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Disclosure Policies of Investment Funds

Abstract

We examine voluntary and mandated disclosure of portfolio holdings by investment funds in a model where funds are characterized as having a stream of investment ideas and as providing liquidity to investors through redemption. We show that the greater is the fund’s liquidity provision role, the more aggressively the fund trades on its ideas, the stronger is its preference to disclose information about its holdings voluntarily, and the weaker is its performance. We also show that mandatory disclosure can appear to increase competition when, in fact, it decreases information available in securities markets by crowding out private information acquisition. Our model provides an explanation for why hedge funds and mutual funds differ in their resistance to public disclosure, and is consistent with stylized facts regarding how funds’ investment decisions respond to poor performance and how differences in disclosure policies affect the future performance of well versus poorly performing funds.
1. Introduction

Even before the financial crisis of 2008 and its aftermath of regulation, significant actions were taken to reshape the disclosure policies of firms in US capital markets. The provisions of Regulation FD and the Sarbanes-Oxley Act were aimed at enhancing the transparency and accuracy of disclosures made by all public companies. The investment management industry, in particular, attracted the attention of regulators with the controversies surrounding late trading and market timing. These practices violated the letter or spirit of rules designed to ensure fairness in dealings between investment funds and their shareholders. Revelations that these practices were widespread and involved both hedge funds and mutual funds led regulators to explore ways to increase the transparency of companies in the investment management industry.

In an effort to monitor better the business practices of hedge funds, the SEC adopted a rule in 2004 requiring most hedge fund advisers to register with the SEC under the Investment Advisers Act. This rule required hedge funds to establish internal compliance systems and to maintain records in accordance with SEC guidelines, and it subjected hedge funds to examination by the SEC. This rule did not require hedge funds to disclose detailed information about their operations or holdings. Nevertheless, memoranda to the SEC during the comment period expressed clear concern that the registration requirement was a first step toward more extensive regulation and oversight.¹ This rule was strongly opposed by the hedge fund industry. It was struck down in 2006 by the U.S. Court of Appeals. In a separate action, the SEC ruled to increase the frequency with which mutual funds must publicly disclose their holdings from semiannually to quarterly. This reversed a 1985 rule change, and brought greater mandatory public disclosure to an area where the degree of disclosure was already relatively high. There seemed to be little if any industry opposition to this rule, however.

Despite the significance of investment management as an area of oversight by regulators, and the relevance of these issues to investors and the investment management

¹One letter quotes Alan Greenspan as saying in Senate testimony that, “should registration fail to achieve the intended objectives, pressure may become irresistible to expand the SEC’s regulatory reach from hedge fund advisers to hedge funds themselves.” See also “Hedge Funds Grow Up: Less Risk, More Regulation—and Lower Returns,” Bloomberg Markets, February, 2005.
industry, little theoretical analysis exists of disclosure policies of investment fund holdings.\(^2\) A great deal of theoretical work has been done on disclosures of product quality or financial condition by sellers of goods or primary security issuers [see Grossman (1981), Diamond (1985), Teoh and Hwang (1991), Shavel (1994), Admati and Pfleiderer (2000), Fishman and Hagerty (1989, 2003, 2004)]. Rich models of insider trading disclosure also exist [see Fishman and Hagerty (1992,1995a), Huddart, Hughes and Levine (2001)]. However, investment funds differ from these types of agents in important ways that make it useful to develop models of disclosure specifically suited to investment funds. In this paper, we model unique aspects of investment funds and draw implications for how a fund’s attributes affect its investment strategy and disclosure policy, and also how these interact.

In our model, investment funds are privately informed similar to insiders. However, two things make funds special. First, their technology depends on generating a stream of investment ideas through time. A fund will typically not know in advance the true payoff to the securities in which it invests. This means a fund might begin accumulating a position that it later learns was a misguided investment, and that it wishes to exit before the uncertainty about its value is resolved. Second, the investors in funds have redemption privileges. Redemptions by investors may force funds to liquidate positions before the investment ideas have run their course.

These aspects of investment funds distinguish them from inside traders who are typically modeled as being fully informed about a security’s payoff up front, and as having the strategic flexibility to trade slowly into a position until the true value of the security is revealed publicly. Unlike insiders, funds might either choose or be forced to liquidate positions at market prices before the uncertainty about a security’s value is fully resolved. Therefore, investment funds need not share the preference of insiders to minimize the price impact of their actions. The possibility of liquidation at an intermediate market price means that disclosures of holdings can enhance an investment fund’s expected profit by moving market prices in the direction of a fund’s holdings.

The things that make investment funds special create disclosure incentives that blend

\(^2\)In fact, we are not aware of any. However, Wermers (2001) contains an excellent discussion of many relevant issues.
those of sellers of goods with those of insiders. If it turns out (ex-post) that a fund chooses or is forced to liquidate at market prices, it faces payoffs that resemble a seller of goods who wants to influence prices in the same direction as its holdings. If not, its incentives are similar to those of an inside trader who wishes to continue accumulating shares with as little price impact as possible. An investment fund selects its disclosure policy to be optimal \textit{ex-ante}, accounting for these (ex-post) possibilities and for how the fund’s trading strategies adapt to the possibilities that can occur.

In a classic paper, Easterbrook and Fischel (1984) argue that disclosure rules in the area of securities regulation have survived largely intact since the 1930s because they coordinate and standardize disclosures that firms would be willing to make voluntarily. So we begin by examining the investment strategies funds follow conditional on both a policy of non-disclosure and a policy of disclosing holdings. Comparing optimized expected profit across policies identifies which policy a fund would adopt voluntarily. We show that the more important are future versus current investment ideas, and the more likely are redemptions, the greater is the value of adopting a liberal disclosure policy. This is because disclosure provides information to the market as a whole that moves prices to levels more favorable to the fund’s liquidation. Alternatively, if future ideas are unlikely to change the fund’s course on an investment, or if the fund can restrain redemptions, it prefers that prices reflect information slowly in order to use a dynamic trading strategy to exploit fully its investment ideas. In this case, secrecy is valuable and the optimal policy is not to disclose holdings. Since both disclosure and aggressive trading move prices, the fund’s optimal choices depend on how disclosure and trading complement each other. We are able to capture these interactions because the trading strategy, disclosure policy and profit are all endogenous in our model.

This discussion suggests why hedge funds and mutual funds differ in the intensity of their opposition to disclosure regulation. Hedge funds utilize “lock-up” and “gate” provisions specifically to prevent untimely redemptions by investors, thus protecting the viability of dynamic trading strategies. Disclosure undermines the profitability of these strategies. In contrast, the vast majority of mutual funds are open-end funds, for which
redemptions can occur at any time.\textsuperscript{3} Liberal disclosure policies can enhance the profitability of open-end funds. Such funds will not utilize fully dynamic trading strategies, however. So in spite of optimizing their trading strategies and disclosure policies to reflect the possibility of redemption, our model predicts that performance will be weaker on average for mutual funds than for hedge funds that are unconstrained by redemption.

Our results indicate that current disclosure rules in the U.S., which do distinguish between mutual funds and hedge funds, are consistent with the private incentives these funds have to disclose their holdings. However, since regulators seem interested in expanding disclosure for hedge funds, we also examine the impact of a disclosure mandate on the investment strategies, profit and the production of costly information by funds. A disclosure mandate will not distort the investment strategies of funds that favor voluntary disclosure. In contrast, we show that imposing a disclosure mandate has a “crowding out” effect on other funds. It eliminates funds from the industry that would not disclose voluntarily and that would be profitable were it not for the disclosure mandate. This reduces industry profit, and eliminates the information those firms would have produced in implementing their investment ideas. A disclosure mandate can therefore result in less information being available to the market as a whole through the system of prices and disclosures despite the mandate to disclose. In fact, a mandate to disclose can distort incentives to such a degree that, for some investment funds, a pure strategy investment rule may not even exist that is consistent with the requirement to disclose.

The perspective that disclosure is linked to redemption has several implications that are consistent with existing empirical evidence. One mentioned above is that hedge funds are superior to mutual funds in their gross-of-fee performance. Others are discussed in detail later in the paper. Perhaps the least obvious is that a liberal disclosure policy has

\textsuperscript{3}The Investment Company Institute reports that at the end of 2003 there was a total of 8124 open-end funds (of which 4598 were equity funds) whose average NAV was $910 million ($800 million). There were only 586 (130 equity) closed-end funds whose average NAV was $402 million ($355 million). Current statistics for the hedge fund industry are difficult to come by (for free). However, the article cited in footnote 1 reports some statistics obtained from Hedge Fund Research, Inc. At the end of Q3 of 2004 there were approximately 7,165 hedge funds worldwide, with a total of $890 billion under management (p. 33). These are managed by 2,750 hedge fund firms, only 160 of which have $1 billion or more under management (p. 39). Thus, for every eight open-end mutual funds there are seven hedge funds; but only 160 hedge fund companies (which might advise multiple hedge funds) have assets under management that approximates the NAV of the average open-end mutual fund.
an asymmetric effect on an investment fund’s performance depending on whether its past performance has been good or poor. In particular, since poor past performers are those for which net redemptions are greatest [see Sirri and Tufano (1998), Sapp and Tiwari (2004) and others], our model predicts that liberal disclosure policies have a positive impact on the future performance of poor past performers. The opposite is predicted for those with good past performance. Ge and Zheng (2006) examine this asymmetry in a panel of mutual funds whose disclosure policies differ in the frequency with which they disclose. These differences exist because many mutual funds voluntarily disclose more frequently than the SEC mandates [see also Elton, Gruber and Blake (2011)]. Controlling for other determinants of fund performance, Ge and Zheng find that performance is indeed positively related to disclosure frequency when past performance has been poor, and negatively related when past performance has been good.

Our contribution to the literature is two-fold. First, we show that the incentives of investment funds for disclosure blend traditional and new considerations in a way that helps explain some empirical observations that existing disclosure models cannot explain. Our second contribution is methodological. Huddart, Hughes and Levine (2001) show that an equilibrium in pure strategies does not exist if the insider in Kyle’s (1985) model is forced to disclose his trading between trading rounds. We model the security market as in Kyle as well. However, pure strategy equilibria exist with disclosure for most parameter combinations describing investment funds in our model. By analyzing a setting that is specialized to investment funds, we are able to identify necessary and sufficient conditions under which pure strategy equilibria exist with disclosure in Kyle models. The key is coupling the possibility of early liquidation with a sequence of signals—precisely the attributes that distinguish investment funds in our model from agents in other models of disclosure.

In our model, the ability of the fund to generate good investment ideas is common knowledge and there is no incentive conflict between fund managers and shareholders. Even in this simple setting, we show that disclosure policy can be an important feature of an investment fund’s operating strategy. Agency conflicts have long been recognized as important in investment management. Screening managers, incentive provision and
monitoring are routinely accomplished through non-public reports to an oversight board, an intermediary or a rating agency. Addressing these issues by private reporting has been modeled in the literature. For example, in Gervais, Lynch and Musto (2005), a family of fund managers is able to screen its members by ability because it observes information that managers cannot credibly communicate to investors. Our study is complementary in demonstrating the importance of public disclosures by investment funds to the security market as a whole.

The paper is organized as follows. The next section presents the model and results. Section 3 discusses robustness. Section 4 discusses empirical implications and existing evidence. Section 5 concludes. Proofs of all propositions appear in the Appendix.

2. Model

We model a single investment fund trading a single risky security. The fund begins with a zero endowment of the security. Figure 1 summarizes the sequence of events. Before trading begins (date 0), the investment fund chooses a disclosure policy regarding its holdings, or recognizes that a mandate to disclose exists. Between dates 0 and 1, the fund observes $v_1$, a signal of the security’s value. At date 1, the fund takes a position in the security. If the disclosure policy the fund chose (or a mandate) requires it, the fund discloses its holdings between dates 1 and 2. The fund then observes $v_2$ and whether or not redemptions will force the fund to liquidate its position. If redemptions do not occur, the fund can trade freely again at date 2 and payoffs occur at date 3 when the security’s true value $v = v_1 + v_2$ is publicly revealed. If redemptions do occur, the fund must liquidate its position at date 2 and the payoff per share is the date-2 market price rather than the security’s true value.\footnote{We assume that if liquidation occurs, the value of the informational advantage associated with having observed $v_2$ is lost; i.e., the investment manager cannot credibly sell the signal. This assumption makes the expected profit function simpler by omitting a term associated with the value of selling $v_2$. However, it does not affect the results because the omitted term would scale expected profit the same regardless of the disclosure policy chosen by the investment fund. See Admati and Pfleiderer (1990) and Fishman and Hagerty (1995b) for analyses of information sales.}

We use a model with two trading dates because it is the simplest setting that captures the idea that disclosure intervenes in the implementation of
an investment strategy, and by doing so affects how the fund trades on its ideas.\footnote{Our focus is on whether the investment fund will commit ex-ante to a policy of disclosing, so we can compare what the fund will do voluntarily to what it would be required to do under a mandate to disclose. This is consistent with what funds do in practice [see Ge and Zhang (2006) and Elton, Gruber and Martin (2011)]. It will always be in the fund’s interest to disclose after learning that redemptions will force liquidation if doing so affects the price in a beneficial manner, and not to disclose otherwise. However, discretion in disclosure undermines its credibility, so a model in which equilibrium exists with credible discretionary disclosure will be quite different from ours. This appears to be commonly known in practice. In pointing out the incentives hedge funds have for selective disclosure, a recent article by the editor of Fortune cautions investors to “Count only the performance numbers that the [hedge fund], in advance, promised to reveal publicly. Disregard anything else.” (Fortune, January 21, 2005, p. 22)}

We refer to \(\{v_1, v_2\}\) as the fund’s stream of investment ideas. The variance ratio \(\phi = \frac{\sigma^2_{v_2}}{\sigma^2_{v_1}}\) captures how investment ideas are distributed across time. Having \(\phi \approx 1\) is probably most relevant for drawing empirical implications. However, we solve the model allowing \(\phi\) to be a free parameter because doing so yields insights about the existence of equilibrium with mandated disclosure. Note that if \(\phi = 0\), the investment fund is fully informed before it trades into its first position in the security, and future information plays no role. In this case, the fund will never wish to reverse course on an investment idea that it has at date 1.

The probability that redemptions force liquidation is denoted by \(q\). In actuality, the propensity of investors to redeem changes through time, so a more realistic approach is to model \(q\) as a random variable whose realization occurs after the investment fund commits to a disclosure policy. We examine this in section 3 below, and the results are qualitatively the same whether we compare high and low values of a predetermined \(q\) or high and low values of the mean of the distribution of a random \(q\). So, for simplicity, we regard \(q\) as a predetermined parameter in the detailed analysis of this section of the paper.

Trading in the security market is modeled as in Kyle (1985). Conditional on its pre-commitment to disclose or not, the investment fund maximizes expected profit by choosing holdings \(x_t\) at each trading date. In each trading round the investment fund’s order is batched for execution with the net orders of liquidity traders \(u_t\). All the orders are then executed by a risk-neutral market maker at a price \(P_t\) equal to the expected value of the security conditional on current and past net order flow and any disclosures that have been made. In accordance with Kyle, we assume that \(\tilde{v}_1 \sim N(0, \sigma^2_{v_1}), \tilde{v}_2 \sim N(0, \sigma^2_{v_2}),\) and \(\tilde{u}_1, \tilde{u}_2 \sim iidN(0, \sigma^2_u)\) are mutually independent and independent of whether redemptions
force liquidation, with $\sigma_{v1} > 0, \sigma_u > 0, \sigma_{v2} \geq 0$ and $0 \leq q \leq 1$. The vector of exogenous parameters $\{\sigma_{v1}, \sigma_{v2}, \sigma_u, q\}$ summarizes the model. Also as in Kyle, we focus on equilibria in which prices are affine functions of variables that the market maker observes.

We assume that whether the investment fund faces redemptions that force liquidation at date 2 is known to the market maker. This ensures that the date-2 order flow is perceived by the market maker as having been drawn from a normal distribution. We also assume that the investment fund starts with sufficient funds to finance its initial investment, and if liquidation does not occur it either has a cash reserve from its initial funding or raises new funds to finance its incremental investment at date 2. If redemptions do force liquidation, we assume that the investment fund must return to investors the entire value of securities held and whatever cash reserve it has. These assumptions are for analytical simplicity. What is essential economically is the possibility of liquidating a significant portion of a profitable investment opportunity. The investment fund’s disclosure and trading decisions are therefore made with an awareness that positions might have to be liquidated prior to the full resolution of the uncertainty upon which the initial investment was based.

\[\text{\textsuperscript{6}}\text{We assume that } \sigma_u \text{ does not depend on the disclosure policy of the investment fund. This means, for example, that the uninformed are not drawn into the market by the precommitment of the investment fund to disclose or by whether there is a mandate to disclose. This assumption makes voluntary disclosure less desirable than it would be otherwise. For example, if liquidity traders are modeled as risk-averse exponential-utility maximizers as in Mendelson and Tunca (2004), then the variance of liquidity trading would likely be greater if the investment fund discloses than if it does not. The impact on our results would be that the voluntary disclosure region in Figure 2 below is larger because disclosing has an added benefit of attracting greater participation by the uninformed.}\]

\[\text{\textsuperscript{7}}\text{Otherwise, the market maker perceives order flow as a draw from a mixture of normals, and we lose the simple information structure afforded by normality. Alternatively, we could assume that the market maker is unaware of redemptions and therefore (counter factually) ignores the possibility that date-2 order flow contains any information about } v_1. \text{ The results would be qualitatively the same, but the investment fund would favor disclosure over a slightly larger parameter space.}\]

\[\text{\textsuperscript{8}}\text{Our model analyzes a single investment position but, in reality, investment funds trade multiple positions simultaneously. When they liquidate to fund redemptions, they ought to choose positions whose liquidation has the smallest impact on expected profit (a delivery option). Though our model ignores the delivery option, our results will still be relevant provided that early liquidation causes investment funds to forgo using a dynamic strategy to exploit some profitable investment ideas. This appears to be true empirically. Alexander, Cici and Gibson (2007) analyze mutual fund trades in response to extreme inflows and outflows. They show that the trades funds make when selling on large outflows involve securities that outperform risk-adjusted benchmarks subsequent to the sale. They attribute this to funds possessing selection ability and having to sell investments with profit potential to finance outflows.}\]
2.1 No Disclosure

In this section, the investment fund neither discloses its trades voluntarily, nor faces a mandate to disclose. We sketch the procedure for solving the model, and we show that an equilibrium exists. We then characterize how the fund’s trading strategy depends on the possibility of early liquidation, and also how this affects the price schedule posted by the market maker.

The fund observes $v_1$ and trades the quantity $\Delta x_1$ at a price of $P_1$ per unit. Order flow is $\omega_1 = \Delta x_1 + u_1$. Between dates 1 and 2 the investment fund (and the market maker) learn whether redemptions will force the fund to liquidate its position or not. The fund also learns $v_2$. If liquidation is not forced, the fund trades $\Delta x_2$ at price $P_2$, and order flow is $\omega_2 = \Delta x_2 + u_2$. However, if liquidation is forced, the fund reverses its date-1 position at price $\hat{P}_2$, and order flow is $\hat{\omega}_2 = -\Delta x_1 + u_2$. Thus, if liquidation is forced, the sequence of order flow conveys two conditionally independent signals of $\Delta x_1$, which depend on $\tilde{v}_1$ alone. If liquidation is not forced, the sequence of order flow contains information about $\tilde{v}_1$ and $\tilde{v}_2$.

The market maker sets prices to the expected value of $\tilde{v}$ conditional on his information. In setting $P_1$, the market maker conditions on $\omega_1$. In setting the date-2 price, he conditions on $\omega_1$ and either $\hat{\omega}_2$ or $\omega_2$ depending on whether redemptions force liquidation or not. Since the market maker knows whether liquidation is forced, he knows whether the functional form of the random order flow he observes at date 2 is $\hat{\omega}_2$ or $\omega_2$. We restrict attention to equilibria in which prices are affine functions of variables the market maker observes:

\[ P_1 = \psi_1 + \lambda_1 \omega_1 \]
\[ P_2 = P_1 + \psi_2 + \lambda_2 \omega_2 \]
\[ \hat{P}_2 = P_1 + \hat{\psi}_2 + \hat{\lambda}_2 (\hat{\omega}_2 + \hat{\theta}_2 \omega_1), \]

where the $\psi, \lambda$ and $\theta$ parameters are determined in equilibrium.

The investment fund’s (random) profit is given by

\[ \tilde{\pi} = \begin{cases} 
(\hat{P}_2 - P_1) \Delta x_1 + 0 & \text{with probability } q \\
(\tilde{v} - P_1) \Delta x_1 + (\tilde{v} - P_2) \Delta x_2 & \text{with probability } 1 - q 
\end{cases} \]

where, despite the absence of tildes, the prices are random variables as well. Since the fund’s initial endowment is zero, its holdings at time 1 are $\Delta x_1$. The first line indicates
that, with probability $q$, the fund will be forced to liquidate its date-1 position at date 2 and carry a zero position thereafter. The second line corresponds to the fund’s profit if liquidation does not occur. The fund maximizes expected profit by choosing $\Delta x_1$ and $\Delta x_2$ conditional on the information it has at the time. We solve this by backward induction.

If liquidation is not forced at date 2, the investment fund chooses its date-2 trade according to

$$\max_{\Delta x_2} E [(\tilde{v} - P_2) \Delta x_2 | v_1, v_2, P_1].$$

Substituting for $P_2$ from equation (1), taking expectations, and rearranging the first-order condition yields

$$\Delta x^*_2 = \delta_2 + \beta_2 (\tilde{v} - P_1) \quad (2)$$

where expressions for $\delta_2$ and $\beta_2$ are given in the equations labeled (A.1) in the Appendix. The second-order condition is satisfied if $\lambda_2 > 0$.

The investment fund’s objective at date 1 is to maximize the expected value of

$$\tilde{\pi}(\Delta x_1) = \begin{cases} (\hat{P}_2 - P_1) \Delta x_1 + 0 & \text{with probability } q \\ (\tilde{v} - P_1) \Delta x_1 + (\tilde{v} - P_2) \Delta x^*_2 & \text{with probability } 1 - q \end{cases} \quad (3)$$

conditional only on $v_1$. After substituting for the $P$s from (1), $\Delta x^*_2$ from (2), and taking expectations conditional on $v_1$, the equation is quadratic in the choice variable $\Delta x_1$. The first-order condition implies

$$\Delta x^*_1 = \beta_0 + \beta_1 v_1,$$

where $\beta_0$ and $\beta_1$ are given in equations (A.5) and (A.6) in the Appendix. The next result verifies that there exists a set of mutually consistent trades that optimize the fund’s expected profit and prices that are equal to the market maker’s conditional expectations of the security’s value.

**Proposition 1.** For any $\sigma_{v_1}$, $\sigma_{v_2}$, $\sigma_u$ and $q < 1$, there exists a unique linear equilibrium in which the investment fund does not disclose its date-1 holdings. Equilibrium parameter values are given by equations (A.1), (A.5)-(A.6), (A.8)-(A.14) in the Appendix.

The price parameters are computed using the Kalman filter, and the results appear as equations (A.8) - (A.14) in the Appendix. These, along with the equations describing
the parameters in the investment fund’s strategy are eleven equations in eleven unknowns. The unique solution to five of the equations is \( \delta_2 = \psi_2 = \hat{\psi}_2 = \beta_0 = \psi_1 = 0 \). The remaining equations can be solved for unique values of \( \lambda_1, \lambda_2, \hat{\lambda}_2, \hat{\theta}_2, \) and \( \beta_2 \) in terms of exogenous parameters and \( \beta_1 \). Finally, \( \beta_1 = h^*_{v} \frac{\sigma_{v1}}{\sigma_{u}} \), where \( h^* \) is defined implicitly as the solution to \( G(h^*; \phi, q) = 0 \). An explicit expression for \( G \) is given in equation (A.20) in the Appendix.

As part of the proof of Proposition 1, we show that \( h^* \) is positive and unique, and that the second-order condition for the investment fund’s date-1 optimization is satisfied if and only if the positive root of \( h^*_{v} \) is selected in defining \( \beta_1 \).

Even though \( h^* \) is defined implicitly, solutions to this model are easy to obtain for any set of exogenous parameters. Given values for \( q \) and \( \phi \), the (unique positive) value of \( h^* \) that satisfies \( G(h^*; \phi, q) = 0 \) can be found via a simple search (e.g., the solver tool in Excel). This, combined with exogenous values for \( \sigma_{v1} \) and \( \sigma_{u} \) determine \( \beta_1 \), which in turn determines the values of all the other endogenous variables.

In a single-period Kyle (1985) model, the insider’s strategy coefficient is simply \( \frac{\sigma_{v1}}{\sigma_{u}} \). Therefore, the leading \( h^*_{v} \) that appears in the solution for \( \beta_1 \) here is a scale factor that incorporates the effects of (i) multiple rounds of trading, (ii) multiple investment ideas, and (iii) the possibility of forced liquidation. These complications are why the solution for \( \beta_1 \) is implicit rather than explicit. Nevertheless, it is possible to provide a detailed characterization of the equilibrium as summarized in the propositions below.

We emphasize two main results. First, the possibilities that \( q > 0 \) and \( \sigma_{v2} > 0 \) each leads investment funds to trade more aggressively both at time 1 and time 2 than if \( q = \sigma_{v2} = 0 \) (i.e., two opportunities to trade on a single signal). Second, the sensitivity of prices to order flow are not monotonic in \( q \) and \( \sigma_{v2} \). The additional aggressiveness of trading can lead to equilibrium prices that are either more or less sensitive to order flow.

It is tempting to think of \( q \) approaching one as analogous to a single-period Kyle (1985) model because the investment fund approaches having only one opportunity to trade on its information. However, this description is incomplete because early liquidation in our model occurs at the date-2 market price whereas liquidation occurs at the security’s true value in a single-period Kyle model. If collapsing to a single trading round were the dominant effect of early liquidation, the investment fund’s trade would converge, as \( q \rightarrow 1 \),
to that of a single-period Kyle insider. This does not happen in our model.

**Proposition 2.** The greater is the probability of liquidation, \( q \), the more aggressively the investment fund trades at date 1.

When \( q \) is small, the investment fund trades less aggressively than in a single-period Kyle (1985) model because there is a high probability that it will be able to trade again at date 2. This is similar to why the insider in a multi-period Kyle model trades less aggressively than the insider in a single-period model. As \( q \) increases, the investment fund increases the aggressiveness of its date-1 trading for two reasons. First, the fund is increasingly likely to have only one opportunity to trade, so restraining itself at date 1 in anticipation of another opportunity to trade at date 2 is less valuable. Second, making its date-1 trade more aggressive increases the information content of the sequence of order flow. This, in turn, increases the precision of the date-2 market price as a forecast of the security’s true value, and raises (lowers) the expected price at which the fund will liquidate a long (short) position if liquidation is forced. This effect is strong enough that, as \( q \) increases, the investment fund trades even more aggressively than if it were the insider in a single-period Kyle model.

It is interesting that even if the fund does not disclose its holdings, aggressive trading serves a purpose similar to disclosure. Specifically, it moves prices in the direction of the fund’s private information, which enhances profit in the event of early liquidation. When we examine later whether a fund will disclose voluntarily, its choice to disclose or not is made knowing that trading more aggressively is available as a means to influence prices. They are not perfect substitutes, however. Trading aggressively provides the fund with the camouflage of liquidity traders, whereas disclosure conveys to the market the fund’s signal \( v_1 \) precisely. In addition, trading aggressively moves the price at which the current trade occurs, whereas disclosure occurs after the fund has established a position in the security and therefore affects only future prices.

The next result shows that \( q \) has an impact on the investment fund’s trading strategy and prices at date 2, even after the uncertainty about liquidation is resolved.

**Proposition 3.** The greater is \( q \), the more aggressively the investment fund trades at date
and the less sensitive is the date-2 price to order flow (in the event that the investment fund does not liquidate at date 2).

If redemptions do not force liquidation at date 2, \( q \) has no direct effect on the fund’s date-2 trade; so Proposition 3 must be understood through the impact of \( q \) on pricing and trading strategies at date 1. Since greater \( q \) leads to more aggressive trading at date 1, less uncertainty about \( \tilde{v} \) exists after date-1 trading, which induces the investment fund to trade more aggressively at date 2. This is because, at date 2, the fund faces a single-period Kyle model. Its strategy coefficient is the ratio of the variance of liquidity trading to the variance of uncertainty about \( \tilde{v} \) faced by the market maker. Less uncertainty implies more aggressive trading. Less uncertainty and more aggressive trading also lead the market maker to set prices that are less sensitive to each unit of order flow.

Propositions 2 and 3 taken together imply that the possibility of early liquidation does not merely shift trading by the fund from date 1 to date 2 or vice-versa, but rather that it increases the overall aggressiveness of trading by the fund. An implication of these results is that investment funds with high probabilities of redemption will not underinvest in securities in order to hold cash to fund redemptions. Instead, the fund will invest more in securities to capitalize on its investment ideas while possessing the resources to do so.

The increase in overall trading by the fund decreases the sensitivity of prices to order flow at date 2. However, the impact on the sensitivity of the date-1 price can be either positive or negative depending on how aggressively the fund trades at date 1, which in turn depends on \( q \) and \( \phi \).

**Proposition 4.** The sensitivity of the date-1 price to order flow is larger for larger \( q \) if and only if  

\[
\frac{q}{1-q} < \frac{3}{2\sqrt{2}} \left( \frac{1}{1+2\phi} \right)^{\frac{1}{2}}.
\]

This result says that unless \( \phi \) is large, equilibrium \( \lambda_1 \) is an inverted-U-shaped function of \( q \). To interpret this, it helps to note that the condition on \( q \) and \( \phi \) in the statement of the proposition is equivalent to \( h_* < 1 \). Recall that when \( h_* = 1 \), the investment fund trades as aggressively as if it were a single-period Kyle (1985) insider. One of the properties of a single-period Kyle model is that the optimal strategy of the insider maximizes the price sensitivity to order flow, where price sensitivity is viewed as a function of the insider’s
strategy coefficient. This is not true in dynamic models, and our model in particular. In a standard dynamic Kyle model (our model with \( q = \phi = 0 \)), the insider’s strategy will always be less aggressive than in a single-period model. In our model, since \( q \) and \( \phi \) vary, the investment fund’s date-1 strategy ranges from being less to more aggressive than in a single-period Kyle model, depending on the values of \( q \) and \( \phi \). The condition in the statement of the proposition reflects the fact that the investment fund trades less aggressively when \( q \) is small than when \( q \) is large.

If \( \phi = 1 \), for example, the point at which price sensitivity at date 1 is maximized is \( q \approx 0.38 \). When \( q \) is small, liquidation is unlikely and the investment fund does not trade aggressively enough to maximize \( \lambda_1 \) because it wants to preserve a portion of its informational advantage for the second period. However, as \( q \) increases, the fund’s investment horizon shortens to a single period and it will be forced to liquidate at a market price that it wants to influence. Both of these factors make the investment fund trade more aggressively, which increases \( \lambda_1 \) to the point where the investment fund trades aggressively enough to maximize \( \lambda_1 \). Beyond that point, further increases in \( q \) lead to even more aggressive trading by the fund, but the information content of those incremental trades is less per unit traded. In this region, \( \lambda_1 \) decreases as \( q \) increases further.

**Proposition 5.** *The more important is the investment fund’s date-2 investment idea (the larger is \( \sigma_{v2} \)), the more aggressively the fund trades at both date 1 and date 2, and the less sensitive is the date-2 price to order flow. The sensitivity of the date-1 price to order flow is larger for larger \( \sigma_{v2} \) if and only if \( \frac{q}{1-q} < \frac{3}{2\sqrt{2}} \left( \frac{1}{1+2\phi} \right)^{\frac{1}{2}} \).*

When the fund anticipates having a large future informational advantage, it “saves” less of its current information for a later trading date, thus trading more aggressively on whatever advantage it has at date 1. If the fund is not required to liquidate at date 2, the greater informational advantage induces it to trade more aggressively at date 2 than if it did not enjoy such an advantage.

The date-2 price is less sensitive for two reasons. First, the more aggressive date-2 trading by the fund means prices are less sensitive to each unit of order flow. Second, the market maker learned more at date 1 because the fund traded more aggressively then as
well. Consequently, the market maker’s beliefs have been updated more regarding \( v_1 \) prior to date 2 than would have been true had \( \sigma_{v_2} \) been smaller. The logic for the sensitivity of the date-1 price is similar to that given following Proposition 4. The value of \( \sigma_{v_2} \) affects whether the investment fund trades less or more aggressively at date 1 through the dependence of \( h_* \) on \( \phi \equiv \frac{\sigma^2}{\sigma_{v_1}} \).

Whether the investment fund voluntarily precommits to disclosing depends on whether disclosing enhances or detracts from its expected profit. The fund’s equilibrium expected profit without disclosure is given in the next result.

**Proposition 6.** If the investment fund does not disclose, equilibrium expected profit is given by

\[
\bar{\pi}_N^* = \sigma_u \sigma_{v_1} N(q, \phi)
\]

where \( N(q, \phi) \) is defined in equation (A.25) in the Appendix.

The leading term \( \frac{\sigma_u \sigma_{v_1}}{2} \) is the insider’s equilibrium expected profit in a standard single-period Kyle (1985) model, so the function \( N(q, \phi) \) captures the impact on expected profit of the possibility of early liquidation, the fund’s ability to follow a dynamic trading strategy, and its having a stream of investment ideas. Since \( h_* \) is chosen to maximize expected profit, the envelope theorem implies that the direct effects of varying \( \phi \) and \( q \) on \( \bar{\pi}_N^* \) prevail in equilibrium. Therefore, from equation (A.25) in the Appendix, \( \bar{\pi}_N^* \) is increasing in \( \sigma_{v_1} \) and \( \sigma_{v_2} \) (i.e., \( \phi \) holding \( \sigma_{v_1} \) fixed) and decreasing in \( q \). Expected profit decreases in \( q \) because liquidation prevents the fund from following a dynamic trading strategy. The fund adjusts to the likelihood of this occurrence by trading more aggressively at date 1 when \( q \) is high, but this only partially offsets the value of the lost opportunity. When \( q > 0 \), in equilibrium the fund bears an expected cost in the form of forgone profit for standing ready to provide liquidity to investors. We now turn to analyzing the case in which the fund discloses its holdings.

### 2.2 Precommitment to Disclose

The setting in which the fund precommits to disclose has to be solved as a dynamic problem as well. However, when an equilibrium exists, the periods decouple because
the disclosure eliminates the investment fund’s advantage at date 2 with respect to the information, $v_1$, that motivates the date-1 trade. This enables us to solve for all endogenous variables explicitly in terms of the exogenous variables. A subtlety is that if both $q$ and $\phi$ are small, an equilibrium in pure strategies will not exist.

To see why not, suppose the market maker believes the investment fund’s date-1 order can be inverted for $v_1$. Then with $\phi$ small, the date-2 price will be insensitive to date-2 order flow because the disclosure gives the market maker almost full information about the value of the security. If $q$ is also small, the fund expects to trade at date 2 rather than liquidate. If the date-2 price is insensitive to order flow, the fund should trade contrary to its information at date 1 to mislead the market maker by its disclosure, planning to follow up with a large trade at date 2 in the direction of its information. This is not an equilibrium because, knowing this, it would not be rational for the market maker to attempt to infer $v_1$ from the disclosure of the fund’s date-1 trade. Alternatively, suppose the market maker believes the investment fund’s disclosure is uninformative about $v_1$. This means the market maker believes the fund’s date-1 trade and hence order flow at date 1 are also uninformative. The date-1 price will be insensitive to date-1 order flow and it would then be in the fund’s interest to place a large order at date 1 in the direction dictated by its information. This is not an equilibrium either. In this case, the fund’s trade and disclosure would be informative about $v_1$, contradicting the supposition that the market maker believes the fund’s disclosure is uninformative.\footnote{There is evidence that mutual funds do trade to manipulate disclosures (i.e., “window dressing”). However, this practice is usually attributed to a fund manager’s desire to mislead the market about the fund’s risk instead of the fundamental value of the securities it holds [see Musto (1999)].}

This insight is attributable to Huddart, et.al. (2001). They incorporate a mandatory disclosure of the insider’s holdings between trading rounds in a standard Kyle (1985) model. They point out that this non-existence problem arises in their setting, which is the “corner” of our model where $\phi = q = 0$. They then construct a mixed-strategy equilibrium in which the insider follows a linear strategy plus noise by adding a normally-distributed random error to an otherwise pure-strategy trade. Adding the error prevents the disclosure from conveying too much information early, which in turn prevents future prices from being
insensitive to order flow. Their result is remarkable because \textit{normally}-distributed noise can achieve this, while preserving the tractability of Kyle’s model.

The conditions that describe agents’ understanding of the environment required of mixed strategies in a Kyle (1985) model are quite strong, however. The insider must mix in a way that elicits a pricing strategy by the market maker that leaves the insider indifferent across all pure-strategy choices. Since the support of the normal distribution is the entire real line, the insider must mix in a very precise way across an infinite number of possibilities between which he is indifferent. At times, he must take large positions on information of little consequence or small positions when his information is significant. Moreover, the market maker is learning through time, so the insider must recalibrate the variance of the distribution of randomization noise in each trading round to reflect the evolution of the market maker’s beliefs; and the market maker must know the variance of the mixing variable at all times. For these reasons, we consider only pure-strategy equilibria in characterizing the situation with disclosure.

In our model, the assumption that the investment fund receives a \textit{stream} of investment ideas prevents too much information about the terminal payoff from being conveyed by the disclosure. We show below that a pure strategy equilibrium does exist with disclosure provided that $\phi > (1 - q)/(1 + q)$. This is not an especially stringent condition. The right-hand side attains its maximum of one at $q = 0$. So as long as $q$ is positive, it suffices that $\phi \geq 1$—the investment idea the fund receives in the second period is at least as important as the one received in the first period.$^{10}$

We now characterize the equilibrium with disclosure. The modification we make to the sequence of events described in subsection 2.1 is that the investment fund discloses its date-1 holdings, $\Delta x_1$, prior to trading at date 2. This means the information set upon which the market maker sets the date-2 price includes $\Delta x_1$ and date-2 order flow.$^{11}$ Since

\footnotesize
$^{10}$The intuition for this is the reverse of the intuition for why an equilibrium fails to exist when $q$ and $\phi$ are small. As long as either $q$ or $\phi$ are not small, the investment fund will want to avoid the destabilizing impact on the date-2 price created by taking a large position at date 1 in an attempt to mislead the market about $v_1$. If the investment fund has to liquidate early, the destabilizing trade will hurt its liquidation proceeds. If $\phi \geq 1$, the second signal is at least as informative as the first, and the fund will regret having misled the market if the second signal contradicts the first.

$^{11}$Conditioning at date 2 on date-1 order flow, $\omega_1 = \Delta x_1 + u_1$, is redundant because only the disclosure $\Delta x_1$
we focus on equilibria in which prices are affine functions of the information the market maker observes, the functional forms are

\[ P_1 = \psi_1 + \lambda_1 \omega_1 \]
\[ P_2 = \alpha_2 \Delta x_1 + \psi_2 + \lambda_2 \omega_2 \]
\[ \hat{P}_2 = \hat{\alpha}_2 \Delta x_1 + \hat{\psi}_2 + \hat{\lambda}_2 (\hat{\omega}_2 + \hat{\theta}_2 \Delta x_1). \]

The solution procedure is the same as in subsection 2.1 above.

**Proposition 7.** A sufficient condition for a unique pure-strategy linear equilibrium to exist with disclosure is \( \phi > (1 - q)/(1 + q) \) (a necessary condition is \( \phi \geq (1 - q)/(1 + q) \)).

In that equilibrium,

\[ P_1 = \left[ \frac{1}{2} (1 - q)^2 \frac{\sigma_{v1}}{\sigma_u} \right] \omega_1 \]
\[ P_2 = \left( \frac{1 - q}{1 + q} \right)^{\frac{1}{2}} \frac{\sigma_{v1}}{\sigma_u} \Delta x_1 + \left[ \frac{\sigma_{v2}}{2\sigma_u} \right] \omega_2 \]
\[ \hat{P}_2 = \left( \frac{1 - q}{1 + q} \right)^{\frac{1}{2}} \frac{\sigma_{v1}}{\sigma_u} \Delta x_1 \]
\[ \Delta x_1^* = \left( \frac{1 + q}{1 - q} \right)^{\frac{1}{2}} \frac{\sigma_u}{\sigma_{v1}} v_1 \]
\[ \Delta x_2^* = \frac{\sigma_u}{\sigma_{v2}} v_2. \]

The investment fund’s expected profit is

\[ \bar{\pi}_D^* = \frac{\sigma_u \sigma_{v1}}{2} D(q, \phi) \]

where \( D(q, \phi) \) is defined in equation (A.41) in the Appendix.

When an equilibrium exists, the disclosure decouples the periods and explicit closed-form solutions are available for all the endogenous variables—no \( G(\cdot) \) as before. If \( q = 0 \), the formulas for the \( \beta \)s and \( \lambda \)s correspond to those of two disjoint single-period Kyle (1985) models where \( v_1 \) is revealed between trading rounds. Economically, this means that disclosure precludes the fund from profiting from dynamic trading on \( v_1 \) even if redemption is informative for \( v \) and \( u_1 \) is not.
does not force liquidation. As shown below, the fund might optimally choose to disclose and make this happen voluntarily.

Larger $q$ leads the investment fund to trade with more urgency at date 1, which in turn reduces the sensitivity of prices to order flow. In fact, for all $q > 0$ the investment fund trades more aggressively than the insider in even a single-period Kyle model. This is because, when the investment fund trades, it accounts for the impact of its trading on the disclosure (in the same way it accounts for its trading on order flow). The larger is $q$, the more desirable is communicating via the disclosure, so the fund trades more aggressively in order to disclose more aggressively. This result is similar to Proposition 2, but the reasoning behind it is different.

Since the periods decouple, if the investment fund is not forced to liquidate at date 2 then its date-2 strategy and the pricing strategy of the market maker are unaffected by the fact that the investment fund might have had to liquidate instead. This is different from the setting without disclosure. There, the possibility of liquidation leads the investment fund to trade more aggressively in both periods, which affects pricing strategies in both periods (Propositions 2 and 3).

The profit function here has the same separability property as the profit function without disclosure in Proposition 6. Both are composed of profit to a single-period Kyle insider, $\frac{\sigma_{v_1}}{2}$, times a scale factor that depends only on $q$ and $\phi$. As in the case with no disclosure, the scale factor here captures early liquidation and the value of the second investment idea $v_2$. However, there is no value to trading on $v_1$ twice with disclosure. Also as before, expected profit with disclosure is increasing in $\sigma_{v_1}$ and $\sigma_{v_2}$, and decreasing in $q$ when a pure-strategy equilibrium exists.

2.3 Voluntary Disclosure

Whether the investment fund voluntarily commits to a policy of disclosing depends on whether its ex-ante expected profit is greater with disclosure than without. Provided that an equilibrium exists, the fund will precommit to voluntary disclosure if and only if $D(q, \phi) > N(q, \phi)$. The separable form of the profit functions and the fact that the existence condition involves only $q$ and $\phi$ imply that whether disclosure is voluntary or
even feasible is not affected by $\sigma_u$, and depends on $\sigma_{v1}$ and $\sigma_{v2}$ only through the ratio $\phi \equiv \frac{\sigma_{v2}^2}{\sigma_{v1}^2}$. Economically, this means that the incentive the investment fund has to disclose does not depend on characteristics of the security market such as the level of camouflage provided by liquidity traders or the scale of the informational asymmetry about the value of the security. What matters are attributes of the investment fund—whether redemption is likely to force liquidation ($q$) and how the fund’s investment ideas are distributed across time ($\phi$). This suggests that the willingness of a fund to disclose depends primarily on its organizational structure and technology rather than attributes of the securities it trades. This means, for example, that those hedge funds ($q \approx 0$) that trade in liquid securities should resist disclosure just as their counterparts that trade in illiquid securities.

Since only two parameters are responsible for whether equilibrium exists with disclosure, and for describing the investment fund’s preference for disclosure, we can easily depict graphically the parameter regions under which the various possibilities occur. Figure 2 displays the values of $D(q, \phi) - N(q, \phi)$ by varying $q$ over $(0, 1)$ and $\phi^{1/2}$ over $[0.2, 2.0]$. The unshaded region covering most of the parameter space is where disclosure is voluntary. The medium shaded region is where an equilibrium exists if disclosure is mandated, but where disclosure by the investment fund is not voluntary. The dark shaded region near the origin is where equilibria exist without disclosure, but no pure strategy equilibrium exists if disclosure is mandated.\(^{12}\)

The region over which an equilibrium does not exist was explained in the previous subsection; it relates to cases in which liquidation is unlikely and the investment fund is nearly fully-informed at date 1. The voluntary disclosure region covers a majority of the rest of the parameter space—i.e., where the probability of forced liquidation is moderate to high. This is because the expected increase in profit from liquidating at a date-2 price that has adjusted more fully to $v_1$ outweighs the expected benefit to trading on $v_1$ dynamically if liquidation does not occur. However, if the probability of forced liquidation

\(^{12}\)Non-existence is not limited to linear equilibria, but applies generally for disclosures that fully communicate what the investment fund knows about $v$. In particular, there cannot exist any equilibrium in which the fund’s disclosure is invertible for $v_1$ when $\phi < (1+q)/(1-q)$. The reason is because if the fund’s disclosure were invertible and there were an equilibrium, prices would have to be linear because conditional expectations are linear in $v_1$. This is not contrary to the Huddart et.al. (2004) result because in their mixed-strategy equilibrium, disclosure is not invertible.
is low, voluntary disclosure will not occur. The expected benefit to trading on \( v_1 \) over two periods outweighs the benefit to having a more informative price in the (low probability) event of liquidation. The range of \( q \) for which disclosure occurs voluntarily expands as \( \phi \) increases because, as the relative importance of \( v_1 \) decreases, so does the value of trading on it dynamically thus reducing the opportunity cost associated with disclosure.

This analysis implies that hedge funds, which constrain investor redemptions, will not voluntarily precommit to disclose. For these funds, the likelihood that redemptions will cause the fund to liquidate positions early is small. This suggests that hedge funds will resist disclosure simply as a consequence of wanting to protect the value of exploiting a dynamic trading strategy. In contrast, open-end mutual funds that have moderate \( q \) will commit voluntarily to a policy of disclosing because they face a greater probability of early liquidation than hedge funds, and disclosure improves the terms on which early liquidation occurs.

In the US, disclosure is mandated for mutual funds but not hedge funds, suggesting that current regulations are designed to serve the divergent private interests of these types of funds.\(^{13}\) The way the SEC implemented the quarterly disclosure rule suggests this also. The rule requires quarterly disclosure of quarterly holdings rather than, say, annual disclosure of quarterly holdings. This would have allowed investors the information needed to verify whether a fund’s investments were consistent with the objectives stated in its offering documents, but would not have served the interest of improving prices at which liquidation occurs. Something akin to annual disclosure of quarter-end holdings, or disclosure with a one-year lag, might achieve the SEC’s potential future objectives for greater transparency in hedge fund operations without impairing the ability of hedge funds to profit from following dynamic strategies.\(^{14}\)

\(^{13}\)Our model does not explain why open-end funds would need regulation in order to enforce disclosure because the single investment fund in our model internalizes the entire benefit to disclosing. In an extended model with multiple investment funds, the funds could well prefer a mandate as a coordination device to control free-rider problems. Without a mandate, funds might withhold disclosure in an attempt to free-ride on the disclosure of other funds who hold the same securities. In this case, an equilibrium with disclosure may not exist even though the funds would collectively be better off if all were to disclose. Easterbrook and Fischel (1984) offer this as a justification for regulatory disclosure mandates. This is modeled formally in Admati and Pfleiderer (2000) for non-financial companies.

\(^{14}\)Disclosure is also mandated for closed-end funds. Our model predicts that this is not consistent with
2.4 Mandatory Disclosure

Our results so far consider the decisions of a fund conditional on its participation in investment management—i.e., conditional on having invested the resources necessary to generate a stream of investment ideas. In this section, we step back and ask how a disclosure mandate affects a fund’s decision to participate. As long as the mandate constraints some funds to disclose that otherwise would not, the mandate reduces profit and potentially causes funds to exit the industry.

To address this, we assume that the fund expends resources of \( c \) to maintain an infrastructure that generates the stream of ideas about the security—e.g., assign an analyst to cover the security, attend conferences, etc. Information will be acquired only if the expected profit net of the information cost is positive.

If disclosure is mandated, the investment fund acquires information if \( \bar{\pi}^*_{D} > c \). If disclosure is not mandated, the fund acquires information if \( \max\{\bar{\pi}^*_{D}, \bar{\pi}^*_{N}\} > c \). If \( c \) is very small (large), the fund acquires (does not acquire) information regardless of whether disclosure is mandated; so the mandate has no effect on information acquisition. However, for \textit{intermediate} values of \( c \), the mandate to disclose can either increase publicly available information about security value by publicizing information that would otherwise have remained private, or crowd out information acquisition by making it unprofitable for some investment funds.

A mandate will affect a fund’s information acquisition decision if and only if investing the private incentives of those funds, which begs the question of why such funds are organized in that manner. One possibility is that closed-end funds begin as hedge funds that subsequently make a public offering, or as open-end funds that decide to discontinue taking further investment. In these cases, it seems conceivable that the investment fund would be willing to bear the cost of the disclosure mandate in order to achieve whatever benefits it perceives to broadening its investor base (in the case of a hedge fund) or capping its size (in the case of an open-end fund). Though closed-end funds can begin as hedge funds, our understanding is that most closed-end funds were registered investment companies (i.e., not hedge funds) from their inception. Our model cannot explain this if organizing as a hedge fund is an option—i.e., if the investment manager can raise sufficient funds while qualifying for an exemption from registration. An explanation that lies outside our model is that the closed-end form is chosen when there are diversification or other advantages to obtaining substantial resources at inception (necessitating a \textit{public} offering and therefore registration) and when the assets the fund plans to hold are sufficiently illiquid that trading to fund redemptions would be very costly for the fund (e.g., real estate, municipal bonds and the like). See Bris, Gulen, Kadiyala and Rau (2007) for an analysis of open-end funds that close.
in information is profitable without a mandate, but not profitable when a mandate exists:

\[ \bar{\pi}^*_N - c > 0 \quad \text{and} \quad \bar{\pi}^*_D - c < 0. \]

In view of the separability properties of these profit functions, this discussion can be summarized as follows.

**Proposition 8.** Assume that an equilibrium with disclosure exists. A mandate to disclose will affect the investment fund’s information acquisition decision, by eliminating information acquisition by the fund, if and only if

\[ D(q, \phi) < \frac{c}{\frac{1}{2}\sigma_u\sigma_v} < N(q, \phi). \]

Recall that disclosure effectively precludes the fund from trading dynamically on its information. The proposition says that a disclosure mandate wipes out what would otherwise be profitable information acquisition in a region of parameter values where the cost of information is high enough that pursuing a dynamic strategy is necessary in order to profit from the information. Investment funds whose viability is impaired by a disclosure mandate are those that trade on information that is profitable only in the absence of disclosure. The competitive advantage of these funds is their ability to exploit over time the information they learn, which is precisely what the disclosure mandate eliminates.

Figure 3 illustrates. The square whose lower-left corner is the origin is the region over which no information is acquired even if a disclosure mandate does not exist. Imposing a mandate on an investment fund in the square has no impact on its information acquisition decision. The area outside the square and above the 45-degree line represents investment funds that disclose voluntarily. Here again, a mandate has no impact on information acquisition. The rectangle to the right of the square and below the 45-degree line is where a mandate eliminates information acquisition by the fund. The area above it, but still to the right of the 45-degree line, is where a mandate would not affect the fund’s information acquisition decision, but would compel an otherwise unwilling fund to disclose its trading. Such an investment fund might be the intended target of a disclosure mandate whose goal is to increase transparency about the manner in which funds operate. However, the
elimination of information acquisition by the unintended targets in the area below has an opposite effect on information about security values. The relative sizes of these areas depend on $c$. Since $D(\cdot)$ and $N(\cdot)$ are both bounded, Figure 3 is a bounded rectangle. If $c$ is high, the area in which imposing a mandate eliminates information acquisition is much larger than the area of unwilling disclosers who continue to acquire information. In this case, imposing a mandate could reduce information available to the public through prices and disclosures.

A more specific perspective is provided in Figure 4, which depicts the impact of a disclosure mandate when $\phi = 1$, for various levels of $\frac{c}{\frac{1}{2}\sigma_u \sigma_v}$ and $q$. The union of all the shaded regions is the set of parameter values under which information will be acquired in the absence of a disclosure mandate. The medium shaded region is the area where disclosure will be voluntary, and the darkest region is the set of parameters for which a disclosure mandate will crowd out information acquisition. As noted above, the information whose acquisition is crowded out is high-cost information at the margin of profitability under voluntary disclosure. An investment fund must follow a dynamic strategy to profit sufficiently from this information to justify its cost. Imposing a mandate in this parameter region decreases information production. Alternatively, where information is inexpensive, a mandate increases information available publicly through prices and disclosures.

This discussion simply highlights the fact that the impact of disclosure mandates on the level of information made public through prices and disclosures is ambiguous. Therefore, regulators interested in enhancing transparency about hedge fund operations through the imposition of disclosure mandates face a tension. The benefits of investor awareness achieved through greater transparency must be weighed against the costs associated with less information acquisition and a less informative price system.

3. Robustness to Uncertainty in the Probability of Liquidation

Thus far, we have assumed that the probability of liquidation is fixed from the time the investment fund chooses its disclosure policy until when it implements its trading strategy. In reality, events that occur after the investment fund chooses its disclosure policy will cause the ex-post redemption probability to be different from what the fund expected. 

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ex-ante. Such events include the fund’s own performance, the performance of the asset class in which the fund invests, and changes in the fund’s management team. Since the disclosure policy choice cannot condition on what that probability will be, the fund has to account for the range of possibilities and optimize in light of how its future investment strategy will adapt to what the redemption probability turns out to be.

To examine this, we consider $q$ to be a random variable whose realization is revealed prior to trading at date 1. The investment strategy at date 1 will adapt to the impact on $q$ of events that occur subsequent to the fund’s disclosure policy choice. However, the fund’s earlier disclosure policy choice must be made based on the fact that $q$ is uncertain, and each possible realization of $q$ has its own optimal trading strategy going forward. For example, when choosing its disclosure policy, the investment fund has to account for the possibilities that it could either retain or lose its star manager. The outcome will affect the propensity of investors to redeem and the likelihood that some investment positions will be liquidated early. The same is true, for example, if the fund invests in a narrow asset class that could lose its appeal with investors as the investment opportunity set changes.

Analytically, the difference between this approach and the model analyzed so far is that $q$ is the realization, prior to date 1, of a random variable $\tilde{q}$ that enters the investment fund’s disclosure policy choice at date 0. The investment fund will commit to disclose if and only if

$$\delta(\phi) \equiv \mathbb{E}_q [D(\tilde{q}, \phi) - N(\tilde{q}, \phi)] > 0.$$  

The functions $D(\cdot)$ and $N(\cdot)$ derived earlier incorporate how market prices and the investment fund’s trading strategy adjust to changes in $q$ prior to date 1. It is clear from Figure 2 that the function $D(q, \phi) - N(q, \phi)$ is concave in $q$ for all $\phi$. Thus, by Jensen’s inequality, uncertainty about $q$ decreases incentives for disclosure. The reason is that sufficient uncertainty means there is enough of a chance the investment fund will not have to liquidate early that it prefers not to disclose.

As an illustration, consider the case of $\phi = 1$ and

$$\tilde{q} = \begin{cases} 
\mu_q + \sigma_q & \text{with probability } 1/2 \\
\mu_q - \sigma_q & \text{with probability } 1/2.
\end{cases}$$

Figure 5 plots $\delta(1)$ for various choices of $\mu_q$ and $\sigma_q$ such that the range of realizations of
are bounded between zero and one. In moving across columns, greater uncertainty, \( \sigma_q \), weakens incentives for voluntary disclosure. Nevertheless, for each level of uncertainty, the incentive for voluntary disclosure increases as \( \mu_q \) increases. This mirrors the conclusion in the subsections above where \( q \) is viewed as a predetermined parameter—that disclosure is favored by funds that face greater expected liquidation. The difference here is that uncertainty about \( q \) increases the cutoff at which disclosure is voluntary.

4. Empirical Implications

The model’s predictions are consistent with several results documented in empirical research that examines hedge funds and mutual funds. First, our model predicts that the gross-of-fee performance of hedge funds should exceed that of mutual funds. Providing liquidity and mandatory disclosure are costly to mutual funds because both limit mutual funds’ ability to exploit dynamic trading strategies. This is consistent with the empirical evidence on differences between mutual fund and hedge fund performance [see Edelen (1999) and Ackerman et.al. (1999)]. It is also consistent with the findings of Bali, Suleyman and Liang (2006) and Aragon (2007) who document significant performance premiums to hedge funds that utilize lock-up provisions versus hedge funds that do not.

The second prediction relates to the predisposition of funds to voluntary disclosure. The structural difference between hedge funds and the vast majority of mutual funds is that hedge funds’ use of lock-up provisions reduces redemption at times that are inopportune to the fund’s investment decisions. Our model predicts that this difference will lead hedge funds to vigorously resist disclosure, whereas mutual funds should resist less or embrace it. Consistent with this, Wermers (1999) documents that many mutual funds did not reduce their disclosure frequency when the SEC changed the reporting mandate from quarterly to semi-annually in 1985. Anecdotal evidence was discussed earlier concerning the differential resistance of hedge funds and mutual funds to recent attempts by the SEC to increase transparency in the hedge fund industry in the U.S.

The third prediction relates to the dependence of trading strategies on the likelihood of redemption. Propositions 2 and 7 indicate that a greater probability of redemption leads the fund to take more aggressive positions on its information regardless of the fund’s
disclosure policy or mandate to disclose. This suggests that regardless of whether a fund is organized as a hedge fund or an open-end mutual fund, the fund should appear to take more risk when it perceives a greater probability of redemptions. This has nothing to do with incentive conflicts, but arises entirely from the fund’s desire to exploit its informational advantage to the greatest extent possible while it has the resources to do so. By not trading aggressively, a fund leaves profit on the table in the event that it expects to face capital constraints later on.

Sirri and Tufano (1992), Sapp and Tiwari (2004) and others document empirically that fund flows follow performance, which implies that the probability of redemptions increase (decrease) when relative performance has been poor (good). Therefore, as a first approximation, poor past performance is a proxy for a heightened probability of redemptions in the near future. According to Propositions 2 and 7, investment funds experiencing poor (good) relative performance should take more (less) aggressive positions. Both mutual and hedge funds should appear to take additional risk to boost returns to “make up for” poor past performance, when in fact, they might be trading to maximize profit given a heightened expectation of impending future redemptions.

This pattern is documented by Brown, Harlow and Starks (1996) for mutual funds and Brown, Goetzmann and Park (2001) for hedge funds. They find that poor relative performers at mid-year increase risk more than good relative performers. Brown, Goetzmann and Park observe that the risk reduction by hedge funds that are performing well is at odds with the incentives to “swing for the fences” embedded in the usual performance-based contracts of hedge fund managers. Their explanation is that hedge fund managers act in a manner consistent with ensuring long-term survival at the expense of failing to maximize short-term compensation. However, this observation is consistent with our model’s prediction as well. If a manager’s forecast of the probability of redemption is low (as it should for a well-performing fund), he takes more time in exploiting the information he has about the true value of the investment position. He will not appear to “swing for the fences.”

Huang, Wei and Yan (2007) explain fund level differences in flow sensitivities to performance as a result of costly search by investors. Our discussion has drawn distinctions between funds with good versus poor performance in determining the likelihood of redemptions. However, their results are a potential source of predictions about what might cause differences in redemption probabilities across funds with similar past
The redemption experience of an investment fund will, of course, often diverge from what was expected when the disclosure policy was chosen. This leads to an asymmetric prediction about how ex-post redemption probabilities affect funds with liberal versus conservative disclosure policies. For example, an investment fund that chooses a liberal disclosure policy might find itself in a situation where the intensity of redemptions is lower than initially anticipated. This could happen if the recent performance of past investment decisions has been especially good. In this case, a liberal disclosure policy actually hurts a fund’s future performance relative to that of other funds with the same past performance but with less liberal disclosure policies. Alternatively, if the intensity of redemption is high, as the investment fund expected when choosing a liberal policy, its choice of disclosure policy enhances its future performance. Thus, a liberal disclosure policy can either detract from or enhance performance depending on whether the intensity of redemption turns out to be low or high.

Again viewing differences in past performance as a primary determinant of differences in the probability of redemption, these observations yield an interesting asymmetric prediction. When past performance is poor (and the redemption probability is high), a liberal disclosure policy is performance-enhancing because disclosure moves prices in a direction more favorable to early liquidation of investment positions. When past performance is good, a liberal redemption policy hurts future performance because disclosure truncates the time the investment fund has to exploit its private information.

Even though mutual funds face disclosure mandates, there is cross-sectional variation in disclosure policies because many funds choose more liberal policies than those mandated by the SEC [see Elton, Gruber and Martin (2011)]. Ge and Zheng (2006) examine these differences empirically by identifying funds that disclose semi-annually, and those that voluntarily precommit to disclosing quarterly either to shareholders, large institutional clients or fund tracking firms. Ge and Zheng analyze the performance of a panel of funds in a regression framework that includes interaction variables between whether the fund discloses quarterly and whether its recent past performance is in the top or bottom quintile of performance for all funds. They document that quarterly disclosure has a positive impact performance.
on relative performance for funds whose past performance is poor and a negative impact for funds whose past performance is good. This finding coincides with the prediction of our model. Moreover, Ge and Zheng show that their result is robust to the inclusion of a variety of other possible determinants of fund performance, and to whether the returns examined are raw or risk adjusted.

Finally, our model predicts what will happen if the regulatory regime changes from one in which investment funds with different liquidation probabilities face different disclosure requirements to a regime where the requirement to disclose is imposed on all funds. For this we need to be explicit about how the model relates to the population of investment funds, however.

Suppose that the single investment fund and the investment cycle in our model depicts one of many iid “draws” of the exogenous parameters, the collection of which forms a panel (cross-section over time) of investment funds. Suppose that funds appear in the panel if their expected profit exceeds the cost of information acquisition, and that management fees are set at competitive levels—i.e., just sufficient to cover information acquisition costs. The current disclosure rules exempt hedge funds from disclosure. By virtue of this exemption, Proposition 8 indicates that some information will be profitable only for hedge funds to acquire. This is because there is a set of “draws” of exogenous parameters (precision and cost of information) for which information acquisition is profitable to hedge funds, but not mutual funds. Such information will be relatively expensive among the information that is acquired across all funds. This implies that hedge funds will have larger fees than mutual funds even if fees are set at competitive levels. However, hedge funds will also have better average gross-of-fee performance than mutual funds because hedge funds benefit from precluding redemption through lock-up provisions. This is consistent with the empirical patterns of fee and performance differences between hedge funds and mutual funds currently.

Proposition 8 implies that the change to a uniform mandate will crowd out acquisition of the costly information that distinguishes hedge funds from mutual funds. This will reduce the profitability of hedge funds because disclosure mandates limit the usefulness of dynamic trading strategies, which will narrow the performance difference between hedge
funds and mutual funds. It will also reduce the fees of hedge funds relative to mutual funds because some hedge funds will no longer be profitable and will exit the industry. These performance differences and fee reductions will not be a consequence of greater competition among investment funds brought about by added transparency. Instead, they reflect the fact that the disclosure mandate causes some otherwise profitable information acquisition and trading to be forgone.

5. Conclusion

We present a model in which an investment fund is characterized as having a stream of investment ideas, and subject to redemptions that lead to early liquidation of investment positions. We analyze the trading strategies, security prices and voluntary disclosure policies that arise in equilibrium, then we examine how a disclosure mandate distorts information acquisition, trading strategies and profit to the investment fund. We show that a fund’s disclosure and trading strategies are critically dependent on the redemption privileges it grants to investors. We discuss the implications of our results for disclosure regulation of hedge funds, which typically restrict investors’ redemption privileges, and mutual funds that typically impose no such restrictions.

We show that if the probability of redemption is low, the investment fund will not voluntarily disclose its holdings. Doing so eliminates the profit associated with pursuing a dynamic versus static trading strategy in exploiting investment ideas. However, if the probability of redemption is moderate to high, the investment fund discloses voluntarily. In this case, the disclosure sufficiently improves the prices at which early liquidation occurs that the fund is willing to forgo the extra profit to following a dynamic strategy. Knowing this, the fund trades more aggressively on its investment ideas when the probability of redemption is high. We also show that when information is costly, a mandate to disclose crowds out what would otherwise be profitable information acquisition by some investment funds that would not voluntarily disclose.

Our results are consistent with the interpretation that existing disclosure rules for mutual funds and hedge funds match their private incentives to disclose holdings. However, our results sound a cautionary note regarding extending to hedge funds disclosure rules
that now apply to mutual funds. Some profitable hedge funds will exit the industry. This crowding out reduces aggregate information acquisition and can, paradoxically, make markets with mandated disclosure less informationally rich than markets without mandated disclosure.

The attributes that describe investment funds in our model are the same as those featured in prominent models of mutual funds. The liquidity provided by mutual funds to their investors through redemption privileges is emphasized in Chordia (1996) and Nanda, Narayanan and Warther (2000). Their work, and that of Berk and Green (2004), also assume that investment companies possess the consistent ability to discover profitable investment ideas.

In our model, the investment fund optimally adjusts its trading and disclosure strategies to account for the possibility of redemption. Despite this, performance is weaker for funds that face a higher probability of redemption. Therefore, in our model, the primary cost of providing liquidity to investors is actually indirect. It is the forgone opportunity to exploit the fund’s private information over time. Empirically, the differences in gross-of-fee performance between mutual funds and hedge funds is large, as are the performance differences between hedge funds that utilize lock-up provisions and those that do not. This suggests that the indirect costs of providing liquidity to investors could be large also.

Our analysis does not consider the possible role that disclosure plays in screening managers, deterring style drift, mitigating incentive problems, or attracting investor participation in markets either directly or through investment funds. Thus, even in a setting where these effects are nonexistent, we show that disclosure policy is an important element of an investment fund’s business strategy, and that regulatory policy concerning disclosure has important and predictable effects on information acquisition, trading strategies and the performance of firms in the investment management business.
APPENDIX

Proof of Proposition 1: Conditional on not being forced to liquidate at date 2, the investment fund solves

$$\max_{\Delta x_2} E[(\tilde{v} - P_2)\Delta x_2|v_1, v_2, P_1].$$

Recognizing that $v = v_1 + v_2$, substituting for $P_2$ from equation (1) and taking expectations yields the equivalent problem

$$\max_{\Delta x_2} (v - P_1 - \psi_2)\Delta x_2 - \lambda_2 \Delta x_2^2.$$

Solving the first-order condition for $\Delta x_2$ yields:

$$\Delta x_2^* = \delta_2 + \beta_2 (v - P_1)$$

where

$$\delta_2 = \frac{-\psi_2}{2\lambda_2} \quad \text{and} \quad \beta_2 = \frac{1}{2\lambda_2}. \quad (A.1)$$

The second-order condition is satisfied when $\lambda_2 > 0$. If the investment fund is forced to liquidate at date 2, $\tilde{\Delta} x_2 = -\Delta x_1$.

At date 1, the investment fund solves

$$\max_{\Delta x_1} E[\tilde{\pi}(\Delta x_1)|v_1]$$

where $\tilde{\pi}(\Delta x_1)$ is given in equation (3). Noting that

$$(v - P_2)\Delta x_2^* = \frac{(v - P_2)(v - P_1 - \psi_2)}{2\lambda_2}$$

$$= \frac{1}{4\lambda_2} (v - P_1 - \psi_2)^2 - \frac{1}{2} (v - P_1 - \psi_2)\sigma_2$$

substituting for the $P$s from equation (1), and taking expectations, we have

$$E[\tilde{\pi}(\Delta x_1)|v_1] = (1 - q) \left\{ (v_1 - \psi_1 - \lambda_1 \Delta x_1)\Delta x_1 + \frac{1}{4\lambda_2} E[(\tilde{v} - P_1 - \psi_2)^2|v_1] \right\}$$

$$+ q \left\{ (\hat{\psi}_2 + \hat{\lambda}_2 (-\Delta x_1 + \hat{\theta}_2 \Delta x_1))\Delta x_1 \right\}. \quad (A.2)$$

Furthermore,

$$E[(\tilde{v} - P_1 - \psi_2)^2|v_1] = v_1^2 + \sigma_v^2 - 2v_1(\psi_1 + \psi_2) + \lambda_1^2 \sigma_v^2 + (\psi_1 + \psi_2)^2$$

$$- 2(v_1 - \psi_1 - \psi_2)\lambda_1 \Delta x_1 + \lambda_1^2 \Delta x_1^2. \quad (A.3)$$
Combining (A.2) and (A.3), and ignoring terms that do not depend on $\Delta x_1$ yields the profit function

$$E[\tilde{\pi}(\Delta x_1)|v_1] = (1 - q) \left\{ (v_1 - \psi_1)\Delta x_1 - \lambda_1 \Delta x_1^2 \right.$$ 
$$+ \frac{1}{4\lambda_2} \left[ \lambda_1^2 \Delta x_1^2 - 2(v_1 - \psi_1 - \psi_2)\lambda_1 \Delta x_1 \right] \right\}$$

$$+ q \left\{ \psi_2 \Delta x_1 + \lambda_2 (\bar{\theta}_2 - 1)\Delta x_1^2 \right\}. \tag{A.4}$$

Solving the first order condition associated with (A.4) for $\Delta x_1$ yields

$$\Delta x_1^* = \beta_0 + \beta_1 v_1$$

where

$$\beta_0 = \frac{q\hat{\psi}_2 + (1 - q) \left\{ \frac{1}{2\lambda_2} (\psi_1 + \psi_2) - \psi_1 \right\}}{2 \left\{ (1 - q)\lambda_1 \left( 1 - \frac{1}{4\lambda_2} \right) + q\lambda_2 (1 - \bar{\theta}_2) \right\}} \tag{A.5}$$

$$\beta_1 = \frac{(1 - q) \left\{ 1 - \frac{\lambda_1}{4\lambda_2} \right\}}{2 \left\{ (1 - q)\lambda_1 \left( 1 - \frac{1}{4\lambda_2} \right) + q\lambda_2 (1 - \bar{\theta}_2) \right\}}. \tag{A.6}$$

The second-order condition is satisfied when

$$(1 - q)\lambda_1 \left( 1 - \frac{\lambda_1}{4\lambda_2} \right) + q\lambda_2 (1 - \bar{\theta}_2) > 0. \tag{A.7}$$

Equilibrium prices are the market maker’s conditional expectations of $\tilde{v}$ given the information he has at the time. At date 1,

$$P_1 = E[\tilde{v}|\omega_1].$$

Since $\omega_1 = \Delta x_1^* + u_1 = \beta_0 + \beta_1 v_1 + u_1$, $\tilde{v}$ and $\tilde{\omega}_1$ are jointly normal random variables. Therefore,

$$E[\tilde{v}|\omega_1] = E[\tilde{v}] + \frac{\text{Cov}[\tilde{v}, \tilde{\omega}_1]}{\text{Var}[\tilde{\omega}_1]} (\omega_1 - E[\tilde{\omega}_1]).$$

Computing moments and simplifying yields

$$P_1 = \psi_1 + \lambda_1 \omega_1,$$

where

$$\psi_1 = -\frac{\beta_0 \beta_1 \sigma_{v_1}^2}{\beta_1^2 \sigma_{v_1}^2 + \sigma_u^2} \tag{A.8}$$

$$\lambda_1 = \frac{\beta_1 \sigma_{v_1}^2}{\beta_1^2 \sigma_{v_1}^2 + \sigma_u^2}. \tag{A.9}$$
If liquidation is not forced at date 2, \( P_2 = E[\tilde{v}|\omega_2, \omega_1] \), or equivalently,
\[
P_2 - P_1 = E[\tilde{v} - P_1 |\omega_2, \omega_1].
\]
Since \( \omega_2 = \Delta x^*_2 + u_2 = \delta_2 + \beta_2(v - P_1) + u_2 \), the random variables \((\tilde{v} - P_1), \omega_2 \text{ and } \omega_1\) are jointly normally distributed. Therefore,
\[
E[\tilde{v} - P_1 |\omega_2, \omega_1] = E[\tilde{v} - P_1 |\omega_1] + \frac{\text{Cov}[\tilde{v} - P_1, \tilde{\omega}_2 |\omega_1]}{\text{Var}[\tilde{\omega}_2 |\omega_1]} (\omega_2 - E[\tilde{\omega}_2 |\omega_1]).
\]
The conditional moments are
\[
\text{Cov}[\tilde{v} - P_1, \tilde{\omega}_2 |\omega_1] = \beta_2 \text{Var}[\tilde{v} - P_1 |\omega_1]
\]
\[
\text{Var}[\tilde{\omega}_2 |\omega_1] = \beta_2^2 \text{Var}[\tilde{v} - P_1 |\omega_1] + \sigma_u^2
\]
\[
E[\tilde{\omega}_2 |\omega_1] = \delta_2 + \beta_2 E[\tilde{v} - P_1 |\omega_1] = \delta_2
\]
\[
\text{Var}[\tilde{v} - P_1 |\omega_1] = \text{Var}[\tilde{v} |\omega_1] = \text{Var}[\tilde{v}] - \frac{\text{Cov}[\tilde{v}, \tilde{\omega}_1]^2}{\text{Var}[\tilde{\omega}_1]}
\]
\[
= \sigma_{v2}^2 + \frac{\sigma_{v1}^2}{\beta_2^2 \sigma_u^2 + 1}.
\]
Substituting and simplifying yields
\[
P_2 = P_1 + \psi_2 + \lambda_2 \omega_2,
\]
where
\[
\psi_2 = -\delta_2 \lambda_2 \tag{A.10}
\]
\[
\lambda_2 = \frac{\beta_2 \text{Var}[\tilde{v} - P_1 |\omega_1]}{\beta_2^2 \text{Var}[\tilde{v} - P_1 |\omega_1] + \sigma_u^2}. \tag{A.11}
\]

If liquidation is forced at date 2, \( \hat{P}_2 = E[\tilde{v}|\hat{\omega}_2, \omega_1] \), or equivalently,
\[
\hat{P}_2 - P_1 = E[\tilde{v} - P_1 |\hat{\omega}_2, \omega_1].
\]
Since \( \hat{\omega}_2 = -\Delta x^*_2 + u_2 = -\beta_0 - \beta_1 v_1 + u_2 \), the random variables \((\tilde{v} - P_1), \hat{\omega}_2 \text{ and } \hat{\omega}_1\) are jointly normally distributed. Therefore,
\[
E[\tilde{v} - P_1 |\hat{\omega}_2, \omega_1] = E[\tilde{v} - P_1 |\omega_1] + \frac{\text{Cov}[\tilde{v} - P_1, \hat{\omega}_2 |\omega_1]}{\text{Var}[\hat{\omega}_2 |\omega_1]} (\hat{\omega}_2 - E[\hat{\omega}_2 |\omega_1]).
\]
The conditional moments are

\[
\text{Cov} \left[ \tilde{v} - P_1, \tilde{\omega}_2|\omega_1 \right] = -\beta_1 \text{Var} \left[ \tilde{v}_1|\omega_1 \right]
\]

\[
\text{Var} \left[ \tilde{\omega}_2|\omega_1 \right] = \beta_1^2 \text{Var} \left[ \tilde{v}_1|\omega_1 \right] + \sigma_u^2
\]

\[
E \left[ \tilde{\omega}_2|\omega_1 \right] = -\beta_0 - \beta_1 E \left[ \tilde{v}_1|\omega_1 \right] = -\beta_0 - \beta_1 P_1
\]

\[
= -\beta_0 - \beta_1 \psi_1 - \beta_1 \lambda_1 \omega_1
\]

\[
\text{Var} \left[ \tilde{v}_1|\omega_1 \right] = \text{Var} \left[ \tilde{v}_1 \right] - \frac{\text{Cov} \left[ \tilde{v}_1, \tilde{\omega}_1 \right]^2}{\text{Var} \left[ \tilde{\omega}_1 \right]}
\]

Substituting and simplifying yields

\[
\hat{P}_2 = P_1 + \hat{\psi}_2 + \hat{\lambda}_2 (\hat{\omega}_2 + \hat{\theta}_2 \omega_1),
\]

where

\[
\hat{\psi}_2 = (\beta_0 + \beta_1 \psi_1) \hat{\lambda}_2
\]

\[
\hat{\lambda}_2 = -\frac{\beta_1 \text{Var} \left[ \tilde{v}_1|\omega_1 \right]}{\beta_1^2 \text{Var} \left[ \tilde{v}_1|\omega_1 \right] + \sigma_u^2}
\]

\[
\hat{\theta}_2 = \beta_1 \lambda_1.
\]

The equations involving \( \psi_1, \psi_2, \beta_0, \delta_2 \) and \( \hat{\psi}_2 \) are five independent linear equations to which the unique solution is \( \psi_1 = \psi_2 = \beta_0 = \delta_2 = \hat{\psi}_2 = 0 \). To solve the remaining equations, begin by rearranging (A.6) as follows

\[
\beta_1 \lambda_1 \left( 1 - \frac{\lambda_1}{4\lambda_2} \right) + \frac{q}{1-q} \beta_1 \hat{\lambda}_2 (1 - \hat{\theta}_2) = \frac{1}{2} - \frac{\lambda_1}{4\lambda_2}
\]

then substitute for \( \hat{\theta}_2 \) from (A.14) and collect like terms

\[
(1 - \beta_1 \lambda_1) \left\{ 1 - \frac{\lambda_1}{4\lambda_2} - \frac{q}{1-q} \beta_1 \hat{\lambda}_2 \right\} = \frac{1}{2}
\]

Note that (A.9) implies

\[
1 - \beta_1 \lambda_1 = \frac{\sigma_u^2}{\beta_1^2 \sigma_{v1}^2 + \sigma_u^2},
\]

(A.13) implies

\[
\beta_1 \hat{\lambda}_2 = \frac{-\beta_1^2 \sigma_{v1}^2}{2 \beta_1^2 \sigma_{v1}^2 + \sigma_u^2},
\]
and (A.11) and (A.1) imply
\[
\lambda_2 \left\{ \beta_2^2 \text{Var} \left[ \bar{v} | \omega_1 \right] + \sigma_u^2 \right\} = \beta_2 \text{Var} \left[ \bar{v} | \omega_1 \right]
\]
\[
\lambda_2 \left\{ \frac{1}{4\lambda_2^2} \text{Var} \left[ \bar{v} | \omega_1 \right] + \sigma_u^2 \right\} = \frac{1}{2\lambda_2} \text{Var} \left[ \bar{v} | \omega_1 \right]
\]
\[
\lambda_2^2 = \frac{1}{4\sigma_u^2} \text{Var} \left[ \bar{v} | \omega_1 \right]
\]
\[
= \frac{1}{4\sigma_u^2} \left\{ \left( \beta_2^2 \sigma_v^2 + \sigma_u^2 \right) \left( \sigma_u^2 \sigma_v^2 + \sigma_u^2 \sigma_v^2 \right) \right\}.
\]

The second-order condition for the investment fund’s date-2 choice is satisfied when \( \lambda_2 > 0 \), so we select the positive root yielding
\[
\frac{1}{\lambda_2} = 2\sigma_u \left\{ \frac{\left( \beta_2^2 \sigma_v^2 + \sigma_u^2 \right)}{\left( \beta_1^2 \sigma_v^2 + \sigma_u^2 \right) \sigma_v^2 \sigma_u + \sigma_v^2 \sigma_u^2} \right\}^{\frac{1}{2}}. \tag{A.18}
\]

Multiplying by \( \frac{1}{4\lambda_1} \) using the expression in (A.9) in place of \( \lambda_1 \) yields
\[
\frac{\lambda_1}{4\lambda_2} = \frac{1}{2} \left\{ \frac{\beta_1^2 \sigma_v^2}{\sigma_u^2} \left( \sigma_v^2 \sigma_u^2 + 1 \right) \right\}^{\frac{1}{2}}. \tag{A.19}
\]

We define \( h \equiv \beta_2^2 \frac{\sigma_v^2}{\sigma_u^2} \) and \( \phi \equiv \frac{\sigma_v^2}{\sigma_u^2} \). Substitute these into (A.16), (A.17) and (A.19), to get
\[
1 - \beta_1\lambda_1 = \frac{1}{h+1}
\]
\[
\beta_1\lambda_2 = \frac{-h}{2h+1}
\]
\[
\frac{\lambda_1}{4\lambda_2} = \frac{1}{2} \left\{ \left( \sigma_v^2 \sigma_u^2 + 1 \right) \right\}^{\frac{1}{2}}.
\]

Substituting these into (A.15) yields
\[
G(h; \phi, q) \equiv \frac{1}{h+1} \left\{ 1 - \frac{1}{2} \left( \frac{h}{(h+1)^2 \phi + (h+1)} \right) \right\}^{\frac{1}{2}} + \frac{q}{1-q} \left( \frac{h}{2h+1} \right) - \frac{1}{2} = 0. \tag{A.20}
\]

This is a necessary condition. If a linear equilibrium exists, then \( \beta_1 \) is given by \( h^* \frac{\sigma_u}{\sigma_v} \), where \( h^* > 0 \) solves \( G(h^*; \phi, q) = 0 \); and values for the remaining endogenous variables are (uniquely) determined by their respective equations above. If the second-order condition for the investment fund’s date-1 choice is satisfied at the values so determined, then those values are an equilibrium.

For any \( \phi \geq 0 \) and \( q \in [0,1) \), at least one solution to \( G = 0 \) exists because \( G \) is continuous, \( G(0; \cdot) = \frac{1}{2} \) and \( \lim_{h \to \infty} G(h; \cdot) = -\frac{1}{2} \). Let \( h^* > 0 \) be such a solution and write (A.6) as
\[
\beta_1 = \frac{(1-q) \left(1 - \frac{\lambda_1}{2\lambda_2} \right)}{S}.
\]
The second-order condition for the investment fund’s date-1 choice is satisfied when $S > 0$. Substituting from the definition of $h$ into (A.19) yields

$$\frac{\lambda_1}{2\lambda_2} = \frac{h_\ast}{(h_\ast + 1)^2 \phi + (h_\ast + 1)} < 1$$

where the inequality is clear by inspection. Thus, $1 - \frac{\lambda_1}{2\lambda_2} > 0$ and hence, the sign of $S$ is the same as the sign of $\beta_1$ in equilibrium. This means that satisfying the second-order condition for the investment fund’s date-1 choice is equivalent to selecting the positive root of $h_\ast$ in determining $\beta_1$ according to $\beta_1 = h_\ast^{\frac{\lambda_1}{\lambda_2}} \frac{\sigma_u}{\sigma_v}$. Taking these facts together implies that (i) any solution to $G = 0$ is an equilibrium provided that its positive root is used in determining $\beta_1$, and (ii) at least one equilibrium exists for any $\phi \geq 0$ and $q \in [0, 1)$.

We now show that, for any $\phi \geq 0$ and $q \in [0, 1)$, equilibrium is unique. This is done by demonstrating that $\frac{\partial G(h_\ast; \phi, q)}{\partial h} < 0$ when evaluated at a solution $h_\ast$. Since $G$ is continuous, this condition implies that $G$ crosses the $h$-axis only once.

To verify, differentiate $G$

$$\frac{\partial G}{\partial h} = -\frac{1}{(h + 1)^2} \{\cdot\} + \frac{1}{h + 1} \{\cdot\}' = \frac{1}{h + 1} \left[\{\cdot\}' - \frac{1}{h + 1} \{\cdot\}\right]$$

where the notation is in obvious analogy to (A.20). At a solution $h_\ast$, $\frac{1}{h + 1} \{\cdot\} = \frac{1}{2}$, so

$$\frac{\partial G}{\partial h} = \frac{1}{h + 1} \left[\{\cdot\}' - \frac{1}{2}\right]. \quad (A.21)$$

Define $f(h) = \frac{h}{(h+1)^2 \phi + (h+1)}$, and note that

$$\{\cdot\}' = -\frac{1}{4} f^{\frac{3}{2}} \frac{\partial f}{\partial h} + \left(\frac{q}{1-q}\right) \frac{1}{(2h+1)^2}.$$

At a solution $h_\ast$,

$$\frac{q}{1-q} = \frac{2h_\ast + 1}{2h_\ast} \left[h_\ast + f(h_\ast)^{\frac{3}{2}} - 1\right]$$

so,

$$\{\cdot\}' = -\frac{1}{4} f^{\frac{3}{2}} \frac{\partial f}{\partial h} + \frac{1}{2h_\ast(2h_\ast + 1)} \left(h_\ast - 1 + f^{\frac{3}{2}}\right).$$

Substituting this and

$$\frac{\partial f}{\partial h} = \frac{(1-h_\ast^2)\phi + 1}{[(h_\ast + 1)^2 \phi + (h_\ast + 1)]^2}$$

into (A.21) and rearranging yields

$$2(h_\ast + 1) \frac{\partial G}{\partial h} = \frac{h_\ast^{\frac{3}{2}} - (1-h_\ast)[(h_\ast + 1)^2 \phi + (h_\ast + 1)]^{\frac{3}{2}}}{h_\ast(2h_\ast + 1)[(h_\ast + 1)^2 \phi + (h_\ast + 1)]^{\frac{3}{2}}}$$

$$- \frac{(1-h_\ast^2)\phi + 1}{2h_\ast^2[(h_\ast + 1)^2 \phi + (h_\ast + 1)]^{\frac{3}{2}}} - 1 \quad (A.22)$$
The sign of $\frac{\partial G}{\partial h}$ is the same as the sign of the right-hand side of (A.22). The right-hand side of (A.22) is negative if and only if
\[
\left\{ h^+ - (1 - h^*)[(h^* + 1)^2 \phi + (h^* + 1)]^{\frac{1}{2}} \right\} 2 \left[ (h^* + 1)^2 \phi + (h^* + 1) \right] - \left[ (1 - h^2) \phi + 1 \right] h^+(2h^* + 1)
\]
\[
2h^*(2h^* + 1) [(h^* + 1)^2 \phi + (h^* + 1)]^{\frac{1}{2}}
\]
is less than one. A tedious calculation shows that this is less than one if and only if
\[
\left[ \frac{h^*}{h^2 + 2h^* \phi + h^* + \phi + 1} \right]^\frac{1}{2} \theta < 2 \left\{ 2h^2 [h^2 \phi + h^* \phi + 1] + \theta \right\}
\]
where $\theta \equiv 2h^3 \phi + 3h^2 \phi + 2h^* \phi + \phi + 1$. Now, $h^* \geq 0, \phi \geq 0, \theta \geq 1$ imply that
\[
\left[ \frac{h^*}{h^2 + 2h^* \phi + h^* + \phi + 1} \right]^\frac{1}{2} \theta < \theta < 2 \theta < 2 \left\{ 2h^2 [h^2 \phi + h^* \phi + 1] + \theta \right\},
\]
which establishes the inequality in (A.23). Conclude, therefore, that $\frac{\partial G}{\partial h}$ is negative in equilibrium, which implies that equilibrium is unique.

**Proof of Proposition 2:** In equilibrium,
\[
G(h^*(q, \phi); q, \phi) = 0
\]
for all $q \in [0, 1]$ and $\phi > 0$. Totally differentiating with respect to $q$ and rearranging yields
\[
\frac{\partial h^*}{\partial q} = -\frac{\frac{\partial G}{\partial q}}{\frac{\partial G}{\partial h}}.
\]
Note that this accounts for adjustments in other endogenous variables to a change in $q$. It is easy to see that $\frac{\partial G}{\partial q} > 0$, and $\frac{\partial G}{\partial h} < 0$ from the uniqueness section of the proof of Proposition 1. Therefore, $\frac{\partial h^*}{\partial q}$ and hence $\frac{\partial \lambda^*_2}{\partial q}$ are positive.

**Proof of Proposition 3:** Let $\beta_t(q)$ denote the equilibrium value of $\beta_t$ as a function of $q$ holding $\phi$ fixed; and similarly for $\lambda_t(q)$. By equation (A.18),
\[
\lambda_2(q) = \frac{1}{2\sigma_u} \left\{ \sigma_v^2 + \frac{\sigma_\epsilon^2 \sigma_u^2}{\beta_1(q) \sigma_v^2 + \sigma_u^2} \right\}^{\frac{1}{2}}.
\]
By Proposition 2, $\beta'_1(q) > 0$, so $\lambda'_2(q) < 0$. By equation (A.1),
\[
\beta_2(q) = \frac{1}{2\lambda_2(q)}
\]
and since $\lambda'_2(q) < 0$, $\beta'_2(q) > 0$. 37
**Proof of Proposition 4:** By equation (A.9),
\[ \lambda_1(q) = \frac{\beta_1(q)\sigma_{v1}^2}{\beta_1^2(q)\sigma_{v1}^2 + \sigma_u^2}. \]

Differentiating
\[ \lambda'_1(q) = \frac{\partial \lambda_1}{\partial \beta_1} \beta'_1(q). \]

Since \( \beta'_1(q) > 0 \), the sign of \( \lambda'_1(q) \) is the same as that of \( \frac{\partial \lambda_1}{\partial \beta_1} \). Now,
\[ \frac{\partial \lambda_1}{\partial \beta_1} = \frac{\sigma_u^2\sigma_{v1}^2 - \beta_1^2\sigma_{v1}^4}{(\beta_1^2\sigma_{v1}^2 + \sigma_u^2)^2} \]

so
\[ \text{Sign} \left\{ \frac{\partial \lambda_1}{\partial \beta_1} \right\} = \text{Sign} \left\{ \sigma_u^2 - \beta_1^2\sigma_{v1}^2 \right\} = \text{Sign} \{1 - h_*\}. \]

From Proposition 2, we know that \( h_* \) is unique and \( G \) is decreasing in \( h \) at \( h_* \). Therefore, \( h_* < 1 \) if and only if \( G(1, \cdot) < 0 \), which is equivalent to
\[ \frac{1}{2} \left\{ 1 - \frac{1}{2} \left( \frac{1}{4\phi + 2} \right) \right\} + \frac{q}{1 - q} \left( \frac{1}{3} \right) - \frac{1}{2} < 0 \]

which simplifies to
\[ \frac{q}{1 - q} < \frac{3}{2\sqrt{2}} \left( \frac{1}{2\phi + 1} \right)^\frac{3}{2}. \]

**Proof of Proposition 5:** The proof mirrors the proofs of Propositions 2 - 4, except that \( \phi \) is varied with \( q \) and \( \sigma_{v1} \) held fixed. The results rest on the fact that \( \frac{\partial G}{\partial \phi} > 0 \) in equilibrium, just as Propositions 2 - 4 rested on the fact that \( \frac{\partial G}{\partial \phi} > 0 \).

**Proof of Proposition 6:** Substituting the equilibrium conditions \( \psi_1 = \psi_2 = \hat{\psi}_2 = 0 \) into equation (A.4) and collecting like terms yields
\[ E[\pi(\Delta x_1)|v_1] = (1 - q) \left\{ \frac{1}{4\lambda_2}(v_1^2 + \sigma_{v2}^2 + \lambda_1^2\sigma_u^2) \right\} \]
\[ + (1 - q) \left( 1 - \frac{\lambda_1}{2\lambda_2} \right) v_1 \Delta x_1 \]
\[ - \left[ (1 - q)\lambda_1 \left( 1 - \frac{\lambda_1}{4\lambda_2} \right) + q\lambda_2(1 - \hat{\theta}_2) \right] \Delta x_1^2. \]

Substituting
\[ \Delta x_1^* = \frac{(1 - q)\left( 1 - \frac{\lambda_1}{2\lambda_2} \right) v_1}{2 \left( (1 - q)\lambda_1 \left( 1 - \frac{\lambda_1}{4\lambda_2} \right) + q\lambda_2(1 - \hat{\theta}_2) \right)}. \]
for $\Delta x_1$ and simplifying yields

$$E[\pi(\Delta x_1^*)|v_1] = (1-q) \left\{ \frac{1}{4\lambda_2} (v_1^2 + \sigma_{v_2}^2 + \lambda_1^2 \sigma_u^2) \right\}$$

$$+ \frac{(1-q)^2 \left( 1 - \frac{\lambda_1}{2\lambda_2} \right)^2 v_1^2}{4 \left[ (1-q) \lambda_1 \left( 1 - \frac{\lambda_1}{4\lambda_2} \right) + q \lambda_2 (1-\hat{\theta}_2) \right]}.$$  

Substituting into the second term using

$$\beta_1 = \frac{(1-q) \left( 1 - \frac{\lambda_1}{2\lambda_2} \right)}{2 \left[ (1-q) \lambda_1 \left( 1 - \frac{\lambda_1}{4\lambda_2} \right) + q \lambda_2 (1-\hat{\theta}_2) \right]}$$

yields

$$E[\pi(\Delta x_1^*)|v_1] = (1-q) \left\{ \frac{1}{4\lambda_2} (v_1^2 + \sigma_{v_2}^2 + \lambda_1^2 \sigma_u^2) + \frac{1}{2} \left( 1 - \frac{\lambda_1}{2\lambda_2} \right) \beta_1 v_1^2 \right\}$$

$$= \frac{(1-q)}{4} \left\{ \left[ \frac{1}{\lambda_2} + 2 \left( 1 - \frac{\lambda_1}{2\lambda_2} \right) \beta_1 \right] v_1^2 + \frac{\sigma_{v_2}^2 + \lambda_1^2 \sigma_u^2}{\lambda_2} \right\}. \quad (A.24)$$

Using the definitions of $h$ and $\phi$, (i.e., $h = \beta_1 \frac{\sigma_u^2}{\sigma_v^2}$ and $\phi = \frac{\sigma_v^2}{\sigma_{v_1}^2}$), equations (A.18), (A.19) and (A.9) are equivalent to:

$$\frac{1}{\lambda_2} = 2\sigma_u \left\{ \frac{h + 1}{(h+1)\phi + 1} \right\}^{\frac{1}{2}}$$

$$\frac{\lambda_1}{2\lambda_2} = \left\{ \frac{h}{(h+1)(h+1)\phi + 1} \right\}$$

$$\lambda_1^2 \sigma_u^2 = \sigma_{v_1}^2 (h+1)^2;$$

respectively. Substituting these back into (A.24) and simplifying yields

$$E[\pi(\Delta x_1)|v_1] = (1-q) \frac{\sigma_u \sigma_{v_1}}{2} (h+1)^{\frac{1}{2}} \left\{ \left[ \frac{h}{h+1} \right]^{\frac{1}{2}} + \left( \frac{1}{1+\phi(h+1)} \right)^{\frac{1}{2}} \left( \frac{1}{h+1} \right) \right\} \frac{v_1^2}{\sigma_{v_1}^2}$$

$$+ \frac{\phi h + (h+1)^{\frac{1}{2}}}{(1+\phi(h+1))^{\frac{1}{2}}}. \quad (A.25)$$

From this equation, and the fact that $h = h^*$ in equilibrium, it is easy to compute the unconditional expectation,

$$\bar{\pi}_N^* = \frac{\sigma_u \sigma_{v_1}}{2} N(q, \phi)$$

where

$$N(q, \phi) \equiv (1-q)(h^*+1)^{\frac{1}{2}} \left\{ \left( \frac{h^*}{h^*+1} \right)^{\frac{1}{2}} + \left( \frac{1}{1+\phi(h^*+1)} \right)^{\frac{1}{2}} \left[ \frac{1}{h^*+1} + \frac{h^*}{(h^*+1)^2} + \phi \right] \right\}. \quad (A.25)$$
Since \( h_\ast \) depends only on \( q \) and \( \phi \), \( N(\cdot) \) is a function of only \( q \) and \( \phi \).

**Proof of Proposition 7:** Conditional on not being forced to liquidate at date 2, the investment fund solves

\[
\max_{\Delta x_2} E \left[ (\tilde{v} - P_2)\Delta x_2 | v_1, v_2 \right].
\]

Recognizing that \( v = v_1 + v_2 \), substituting for \( P_2 \) from equation (4) in the text and taking expectations yields the equivalent problem

\[
\max_{\Delta x_2} (v - \alpha_2 \Delta x_1 - \psi_2)\Delta x_2 - \lambda_2 \Delta x_2^2.
\]

Solving the first-order condition for \( \Delta x_2 \) yields:

\[
\Delta x_2^\ast = \delta_2 + \beta_2 (v - \alpha_2 \Delta x_1)
\]

where

\[
\delta_2 = -\frac{\psi_2}{2\lambda_2} \quad \text{and} \quad \beta_2 = \frac{1}{2\lambda_2}. \tag{A.26}
\]

The second-order condition is satisfied when \( \lambda_2 > 0 \). If the investment fund is forced to liquidate at date 2, \( \Delta x_2 = -\Delta x_1 \).

At date 1, the investment fund maximizes the expected value of

\[
\tilde{\pi}(\Delta x_1) = \begin{cases} 
(\tilde{v} - P_1)\Delta x_1 + (v - P_2)\Delta x_2^\ast & \text{with probability } (1 - q) \\
(\tilde{v} - P_1)\Delta x_1 + 0 & \text{with probability } q
\end{cases}
\]

conditional on \( v_1 \). After substituting for \( \Delta x_2^\ast \) and the functional forms of the prices from equation (4) in the text, completing squares, and taking the conditional expectation yields:

\[
E [\tilde{\pi}(\Delta x_1)|v_1] = (1 - q) \left\{ (v_1 - \psi_1)\Delta x_1 - \lambda_1 \Delta x_1^2 
+ \frac{1}{4\lambda_2} \left[ v_1^2 + \sigma_{v_2}^2 - 2v_1 \alpha_2 \Delta x_1 - 2v_1 \psi_2 + (\alpha_2 \Delta x_1 + \psi_2)^2 \right] \right\}
+ q \left\{ (\hat{\alpha}_2 - \hat{\lambda}_2 (1 - \hat{\theta}_2) - \lambda_1) \Delta x_2 + (\hat{\psi}_2 - \psi_1) \Delta x_1 \right\}. \tag{A.27}
\]

Solving the first-order condition for \( \Delta x_1 \) yields:

\[
\Delta x_1^\ast = \beta_0 + \beta_1 v_1
\]

where

\[
\beta_0 = \frac{(1 - q) \left( \frac{\psi_2}{2\lambda_2} \alpha_2 - \psi_1 \right) + q \left( \hat{\psi}_2 - \psi_1 \right)}{(1 - q) \left( 2\lambda_1 - \frac{\alpha_2^2}{2\lambda_2} \right) - 2q \left[ \hat{\alpha}_2 - \hat{\lambda}_2 (1 - \hat{\theta}_2) - \lambda_1 \right]},
\]

\[
\beta_1 = \frac{(1 - q) \left( \frac{\alpha_2^2}{2\lambda_2} \right) - 2q \left[ \hat{\alpha}_2 - \hat{\lambda}_2 (1 - \hat{\theta}_2) - \lambda_1 \right]}{(1 - q) \left( 2\lambda_1 - \frac{\alpha_2^2}{2\lambda_2} \right) - 2q \left[ \hat{\alpha}_2 - \hat{\lambda}_2 (1 - \hat{\theta}_2) - \lambda_1 \right]}. \tag{A.28}
\]
The second-order condition simplifies to

\[ \lambda_1 + q \hat{\lambda}_2 \left( 1 - \hat{\theta}_2 \right) - \left( 1 - q \right) \frac{\alpha^2}{4\lambda_2} + q \hat{\alpha}_2 > 0. \] (A.29)

Since the \( \lambda \) (\( \alpha \)) parameters measure the sensitivity of prices to order flow (the disclosure of \( \Delta x_1 \)), this condition means that the market maker’s reliance on order flow in drawing an inference about \( \hat{v} \) cannot be small relative to his reliance on the disclosure.

Reasoning exactly as in Proposition 1,

\[ P_1 = \psi_1 + \lambda_1 \omega_1 \]

where

\[ \psi_1 = -\frac{\beta_0 \beta_1 \sigma_x^2}{\beta_1^2 \sigma_{v1}^2 + \sigma_u^2} \] (A.30)

\[ \lambda_1 = \frac{\beta_1 \sigma_x^2}{\beta_1^2 \sigma_{v1}^2 + \sigma_u^2}. \] (A.31)

Though the parametric form of \( P_1 \) looks the same as in Proposition 1, the values of the endogenous variables will generally not be the same as in the setting without disclosure even if the values of the exogenous variables are the same.

If liquidation is not forced at date 2, \( P_2 = E[\hat{v}|\omega_2, \omega_1, \Delta x_1^*] \). The conditioning variable \( \Delta x_1^* \) is included because we are now working under the assumption that the investment fund has either precommitted or is under a mandate to disclose. Since \( \omega_2 = \Delta x_2^* + u_2 = \delta_2 + \beta_2 (v - \alpha_2 \Delta x_1^*) + u_2 \) and \( \Delta x_1^* = \beta_0 + \beta_1 v_1 \), all four random variables are jointly normally distributed. Therefore,

\[ E[\hat{v}|\omega_2, \omega_1, \Delta x_1^*] = E[\hat{v}|\omega_1, \Delta x_1^*] + \frac{Cov[\hat{v}, \omega_2|\omega_1, \Delta x_1^*]}{\text{Var}[\omega_2|\omega_1, \Delta x_1^*]} (\omega_2 - E[\omega_2|\omega_1, \Delta x_1^*]). \]

If the investment fund’s second-order condition at date 1 is satisfied, then its optimal strategy is an affine function of \( v_1 \) and the market maker can invert the disclosure of \( \Delta x_1^* \) for \( v_1 = \frac{1}{\beta_1} (\Delta x_1^* - \beta_0) \).

Therefore, the conditional moments are

\[ E[\hat{v}|\omega_1, \Delta x_1^*] = \frac{1}{\beta_1} (\Delta x_1^* - \beta_0) \]

\[ \text{Cov}[\hat{v}, \omega_2|\omega_1, \Delta x_1^*] = \beta_2 \sigma_{v1}^2 \]

\[ \text{Var}[\omega_2|\omega_1, \Delta x_1^*] = \beta_2 \sigma_{v2}^2 + \sigma_u^2 \]

\[ E[\omega_2|\omega_1, \Delta x_1^*] = \delta_2 + \beta_2 \left[ \frac{1}{\beta_1} (\Delta x_1^* - \beta_0) - \alpha_2 \Delta x_1^* \right]. \]

Substituting and collecting like terms yields

\[ P_2 = \alpha_2 \Delta x_1 + \psi_2 + \lambda_2 (\omega_2 - \theta_2 \Delta x_1) \]
where

\[ \alpha_2 = \frac{1}{\beta_1} \]  \hspace{1cm} (A.32)
\[ \psi_2 = -\frac{\beta_0}{\beta_1} \left( 1 - \frac{\beta_2^2 \sigma_{v_2}^2}{\beta_2^2 \sigma_{v_2}^2 + \sigma_u^2} \right) - \frac{\beta_2^2 \sigma_{v_2}^2}{\beta_2^2 \sigma_{v_2}^2 + \sigma_u^2} \delta_2 \]  \hspace{1cm} (A.33)
\[ \lambda_2 = \frac{\beta_2 \sigma_{v_2}^2}{\beta_2^2 \sigma_{v_2}^2 + \sigma_u^2} \]  \hspace{1cm} and \hspace{1cm} \theta_2 = 0. \]  \hspace{1cm} (A.34)

If liquidation is forced at date 2, \( \hat{P}_2 = E[\hat{v}|\hat{\omega}_2, \omega_1, \Delta x^*_1] \), where \( \hat{\omega}_2 = -\Delta x^*_1 + u_2 \). Since all the variates are jointly normal,

\[ E[\hat{v}|\hat{\omega}_2, \omega_1, \Delta x^*_1] = E[\hat{v}|\omega_1, \Delta x^*_1] + \frac{\text{Cov}[\hat{v}, \hat{\omega}_2|\omega_1, \Delta x^*_1]}{\text{Var}[\hat{\omega}_2|\omega_1, \Delta x^*_1]} (\hat{\omega}_2 - E[\hat{\omega}_2|\omega_1, \Delta x^*_1]). \]

Assuming the investment fund’s second-order condition is satisfied, \( \Delta x^*_1 \) can be inverted for \( v_1 \) and the conditional moments are

\[ E[\hat{v}|\omega_1, \Delta x^*_1] = \frac{1}{\beta_1} (\Delta x^*_1 - \beta_0) \]
\[ \text{Cov}[\hat{v}, \hat{\omega}_2|\omega_1, \Delta x^*_1] = \text{Cov}[v_1 + \bar{v}_2, -\beta_0 + \beta_1 v_1 + \bar{u}_2|\omega_1, \Delta x^*_1] = 0. \]

Therefore,

\[ \hat{P}_2 = \hat{\alpha}_2 \Delta x_1 + \hat{\psi}_2 + \hat{\lambda}_2 (\omega_2 - \hat{\theta}_2 \Delta x_1) \]

where

\[ \hat{\alpha}_2 = \frac{1}{\beta_1} \]  \hspace{1cm} (A.35)
\[ \hat{\psi}_2 = -\beta_0 \]  \hspace{1cm} (A.36)
\[ \hat{\lambda}_2 = \hat{\theta}_2 = 0. \]  \hspace{1cm} (A.37)

The equations involving \( \delta_2, \beta_0, \psi_1, \psi_2 \) and \( \hat{\psi}_2 \) are five independent linear equations whose unique solution is the zero vector. Combining equations (A.26) and (A.34) and solving yields

\[ \lambda_2 = \frac{\sigma_{v_2}}{2 \sigma_u} \]  \hspace{1cm} (A.38)
\[ \beta_2 = \frac{\sigma_u}{\sigma_{v_2}}. \]  \hspace{1cm} (A.39)

Substituting from (A.31), (A.32), (A.35), (A.37) and (A.38) into equation (A.28) and simplifying yields

\[ \beta_1 = \left( \frac{1 + q}{1 - q} \right)^{\frac{1}{2}} \frac{\sigma_u}{\sigma_{v_1}} \]
which implies

\[ \hat{\alpha}_2 = \alpha_2 = \left( \frac{1 - q}{1 + q} \right)^{\frac{1}{2}} \sigma_v \frac{\sigma_1}{\sigma_u} \]

\[ \lambda_1 = \frac{1}{2} \sigma_v (1 - q^2)^{\frac{1}{2}} \]

where the signs of \( \hat{\alpha}_2, \alpha_2 \) and \( \lambda_1 \) are all the same as that of \( \beta_1 \), which depends on whether the positive or negative root of \( \frac{1 + q}{1 - q} \) is selected in computing \( \beta_1 \).

Since \( \hat{\lambda}_2 = \hat{\theta}_2 = 0 \) in equilibrium, the investment fund’s date-1 second-order condition (A.29) reduces to

\[ \lambda_1 - q\hat{\alpha}_2 - (1 - q) \frac{\alpha_2^2}{4\lambda_2} > 0 \]  
(A.40)

If the positive root is selected in computing \( \beta_1 \), then \( \lambda_1 \) and \( \alpha_2 \) will also be positive. Substituting those values and \( \alpha_2 \) and \( \lambda_2 \) into (A.40) reduces to

\[ 1 - \left( \frac{1 - q}{1 + q} \right)^{\frac{1}{2}} \sigma_v \frac{\sigma_1}{\sigma_v} > 0 \]

where \( \left( \frac{1 - q}{1 + q} \right)^{\frac{1}{2}} \) denotes the positive root. This is equivalent to

\[ \phi > \left( \frac{1 - q}{1 + q} \right) \]

which verifies that this condition is sufficient for a pure strategy equilibrium in which the positive root is selected in computing \( \beta_1 \). Similar reasoning implies that \( \phi \geq (1 - q)/(1 + q) \) is necessary.

Alternatively, if the negative root is selected in computing \( \beta_1 \), then \( \lambda_1 \) and \( \alpha_2 \) will also be negative. Substituting those values and \( \alpha_2 \) and \( \lambda_2 \) into (A.40) reduces to

\[ -1 - \left( \frac{1 - q}{1 + q} \right)^{\frac{1}{2}} \sigma_v \frac{\sigma_1}{\sigma_v} > 0 \]

which is false for all \( q \in [0, 1] \), \( \sigma_v_1 > 0 \) and \( \sigma_v_2 \geq 0 \), so the second-order condition cannot hold. Therefore, \( \beta_1 < 0 \) cannot be true in equilibrium.

To compute equilibrium expected profit, collect terms of like orders of \( \Delta x_1 \) in equation (A.27) to get

\[
E[\tilde{\pi}(\Delta x_1)|v_1] = (1 - q) \left[ 1 - \frac{\alpha_2}{2\lambda_2} \right] v_1 \Delta x_1 - \left[ \lambda_1 - (1 - q) \frac{\alpha_2^2}{4\lambda_2} - q\hat{\alpha}_2 \right] \Delta x_1^2 \\
+ (1 - q) \frac{1}{4\lambda_2} \left( v_1^2 + \sigma_v^2 \right).
\]
Substituting for $\Delta x_1 = \beta_1 v_1$, $\alpha_2 = \hat{\alpha}_2 = 1/\beta_1$, $\lambda_1 = \frac{1}{2\beta_1} (1+q)$ and $\lambda_2 = \frac{\sigma_{v1}}{2\sigma_u}$, taking the expectation over $v_1$ and simplifying yields

$$\bar{\pi}_D = \frac{\sigma_{v1}^2}{2} (1 - q) \left\{ \beta_1 + \frac{\sigma_u \sigma_{v2}}{\sigma_{v1}^2} \right\}.$$  

Substituting for $\beta_1 = \left( \frac{1+q}{1-q} \right)^{\frac{1}{2}} \frac{\sigma_u}{\sigma_{v1}}$ and $\phi^{\frac{1}{2}} = \frac{\sigma_{v1}}{\sigma_{v2}}$ yields

$$\bar{\pi}_D = \frac{\sigma_u \sigma_{v1}}{2} D(q, \phi)$$

where

$$D(q, \phi) \equiv (1 - q) \left\{ \left( \frac{1+q}{1-q} \right)^{\frac{1}{2}} + \phi^{\frac{1}{2}} \right\}.$$  \hspace{1cm} (A.41)

This is easily shown to be decreasing in $q$ for all $\phi$ for which a pure strategy equilibrium exists—i.e., $\phi \geq \left( \frac{1-q}{1+q} \right)$. 

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Figure 1
Timeline of Events

- Fund chooses disclosure policy or faces mandate
- Fund observes $v_1$
- Fund trades to holdings of $x_1$
- Fund discloses $x_1$ or not according to policy or mandate
- Fund observes $v_2$ and learns whether redemptions require liquidation
- Fund either liquidates $x_1$ to fund redemptions, or trades to $x_2$
- Security pays off $v = v_1 + v_2$

- date 0
- Trading date 1
- Trading date 2
- Payoff date
**Figure 2**

Voluntary Disclosure Region when Information is Costless

\[ D(q, \phi) - N(q, \phi) \]

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*unshaded is the voluntary disclosure region*

*no equilibrium exists in pure strategies*

*grey is the no disclosure region*
Figure 3
Impact of Disclosure Mandate on Information Acquisition

\[ D(q, \phi) = N(q, \phi) \]

- Disclosure voluntary
- Mandate irrelevant

- No info acquired
- Mandate irrelevant

- Info acquisition survives mandate

- Info acquisition crowded out by mandate

\[ \frac{c}{0.5 \sigma_u \sigma_v} \]
### Figure 4  
**Acquisition of Costly Information when \( \phi = 1 \)**  
\[
\text{max\{D, N\}} - \frac{c}{0.5 \sigma_u \sigma_v} 
\]

**C / (0.5 \sigma_u \sigma_v)**

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- **costly information is not acquired even without a mandate**
- **costly information is acquired even with a mandate**
- **disclosure is voluntary - mandate irrelevant**
- **mandate crowds out information acquisition**
### Figure 5

**Voluntary Disclosure Region if q is Uncertain Ex-Ante**

\[ E[D(q,\phi) - N(q,\phi)] \]

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- unshaded is voluntary disclosure region
- shaded is no voluntary disclosure region
REFERENCES


