Explaining Momentum within an Existing Risk Factor Model

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November, 2012

Abstract

I propose that abnormal returns generated by price momentum can be explained within the framework of an existing risk factor model such as the Fama-French three-factor model. Two features of a systematic factor, weakly positive autocorrelation and the leverage effect, generate a small positive alpha in the factor portfolio scaled by its own past returns. The momentum portfolio magnifies this alpha by taking long positions in stocks with highly positive (negative) betas and short positions in stocks with highly negative betas given a positive (negative) realized factor return. Time-varying stock betas enhance the degree of magnification significantly. I demonstrate that a simulated market in which asset returns obey the CAPM or the three-factor model can produce realistic momentum dynamics and substantial abnormal profits. In empirical tests, I show that a replicating portfolio with time-varying betas accounts for 50% of the Fama-French alpha of the canonical momentum portfolio and 75% of the value-weighted momentum portfolio. Among firms larger than the NYSE median, the momentum strategy is no longer profitable after taking into account the dynamic replicating portfolio. The residual alpha can be attributed to small firms suffering recent losses and can be completely explained away with the addition of a financial distress factor.

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1 Introduction

Price momentum is a well-documented stock market phenomenon in which stocks with high recent returns ("winners") tend to outperform those with low recent returns ("losers") in subsequent months. The momentum portfolio that buys winners and short-sells losers based on stock returns within the past year is held for up to one year forward. It is a persistent and puzzling fact that such a zero-investment portfolio generates an abnormal positive profit, which is highly significant even after controlling for established systematic risk factors. Since it was first described in Jegadeesh and Titman (1993), momentum has become somewhat of an enigma and its genesis a hotly debated topic. Fama and French (1993, 1996) successfully explained a large portion of the cross-sectional variation in stock returns using a three-factor model (henceforth FF3) consisting of market (MKTRF), size (SMB) and value (HML); however, they had to concede that their model was unable to explain momentum. I will argue that the Fama-French three-factor model *can* in fact account for a large portion of momentum's abnormal returns, on its own, without any additional assumptions or theories.

My explanation for momentum is a mechanical one where the observed positive alpha from the unconditional factor regression is not real but arsies due to mismeasurement of factor betas. The momentum portfolio does not have constant loadings on risk factors but rather highly time-varying ones that depend on past realizations of factor returns. When the factors exhibit positive autocorrelation and leverage effect (past returns negatively predicting squared future returns), momentum inherits and amplifies these predictability features. Such features, in turn, cause the unconditional factor regression model to generate an artificial alpha, even if there is actually none. This mechanism is powerful in that only a small amount of predictability in the risk factors is needed to generate a large unconditional alpha in the momentum portfolio; it is also universal in that it works for any set of assets that obeys a linear risk factor model.

Assuming that the Fama-French three-factor model is true, I can construct a replicating portfolio for momentum. It is a portfolio of risk factors scaled by a weighted average beta of firms chosen into the momentum portfolio; the weight assigned to each firm beta is equal to the weight that momentum assigns to its returns. With this replicating portfolio, I can explain away 84% of the mean return and 75% of the FF3 alpha of the value-weighted momentum portfolio. The residual return and alpha are statistically indistinguishable from zero. This result is significant because it demystifies momentum within the confines of an existing and well-accepted multifactor pricing model. Among small firms in which momentum is not as well explained by the replicating portfolio, the addition of a financial distress factor can close the gap. The empirical evidence I will present below paints a complete picture of momentum within a parsimonious framework.

It is important to understand the exact mechanism that gives rise to momentum alpha and identify the parameters that determine its magnitude. The mechanism can be divided into two parts, one that links momentum betas to past factor returns and the other that links factor predictability to alpha. At each time the portfolio is formed, stocks are sorted based on their recent returns, which are determined by their systematic returns (the realized factor returns times individual firm betas on those factors) and idiosyncratic returns. Given that a factor has a positive realized return, winners are more likely to be firms with high betas on that factor, while losers are more likely to be ones with low betas. The momentum portfolio would then have a highly positive beta on the factor. The opposite is true given a negative realized factor return. The magnitude of the realized factor return also determines the extent to which a firm's beta determines its rank in the return sort. Therefore, momentum's portfolio beta on that factor is time-varying and proportional to the realized factor returns.

An unconditional factor regression specification is required to assign constant betas to a portfolio with time-varying betas. The beta estimates are roughly equal to the average portfolio betas over the sample period, so if the time-varying betas are positively correlated with factor returns, the regression gives a positive intercept estimate, or alpha, since $E[wF] - E[w]E[F] = \text{cov}(w, F)$. In addition, if the portfolio beta is negatively correlated with the squared factor returns, there is an additional source of positive alpha. I will show that these two conditions are indeed satisfied in the data and are responsible for creating a large positive alpha in momentum.

Momentum betas inherit the ability to predict factor returns and return volatilities from features that already exist within the factor structure. The Fama-French three factor returns, for example, exhibit varying degrees of positive autcorrelation and leverage effect. Even though the strength of the correlation is weak, the magnitude is statistically significant. Momentum takes full advantage of this feature and magnifies it to a great degree. Since only stocks at the extreme ends of the return sort are selected into the momentum portfolio, its beta on a risk factor is a large multiple of the past factor returns.

The size of the multiplier depends on the cross-sectional dispersion of individual firm betas and the size of the idiosyncratic volatility. In simulation, increasing the former and reducing the latter both lead to increases in momentum alpha. With values calibrated from real data, the simulated market produces a momentum portfolio that is very similar to the dynamic replicating portfolio in the empirical analysis, particularly in terms of FF3 alpha and correlation with the actual momentum portfolio. This experiment, along with the empirical reconstruction of momentum profits from the factor-level correlations, provides a comprehensive analysis of the mechanism that I described and proves that it works as intended in both a controlled environment and historical data.

In previous literature, the work most closely related to mine is that of Grundy and Martin (2001), who adjusted the momentum portfolio by time-varying exposures to common risk factors and found that the adjustment significantly reduced variability of returns. They also found that momentum profit and alpha *increased* after controlling for time-varying betas. Chordia and Shivakumar (2002) cited this conclusion but raised doubt about its validity. They used a set of lagged macroeconomic variables relating to the business cycle to predict one-month-ahead returns. The predicted part of returns can largely explain momentum profits, giving hope that a conditional pricing model may yet prove a viable path. My results are consistent with Chordia and Shivakumar's but require no macroeconomic variables as predictors.

At the same time, my results run counter to Grundy and Martin's for a number of reasons. One is that they defined momentum by equally-weighting top and bottom deciles of stocks sorted on past returns, so that the portfolio is dominated by a few very small firms. I examine the entire cross-section and show that the replicating portfolio works well for most firms except the very small. Another difference is that Grundy and Martin estimated pre-formation firm betas using a five-year window, which systematically underestimates the magnitude of the momentum portfolio betas. I will show that individual firms have time-varying betas affected by persistent shocks, and a short window of two years leads to much more accurate estimates. GM also estimated post-formation firm betas using a rather unusual five-month window and constructed an ex post replicating portfolio that yields a negative alpha. I repeat the same exercise using a two-year estimation window and find that the ex post replicating portfolio has a highly positive FF3 alpha, similar to the ex ante replicating portfolio.

Most existing explanations for momentum fall into one of two categories: behavioral or rational. The behaviorists argue that price continuation is irrational and is the result of cognitive biases such as under-reaction. Barberis, Shleifer and Vishny (1998) and Hong and Stein (1999) both offer models of investors' bounded rationality that leads to under-reaction to news. The former has investors suffer from representativeness heuristics of Tversky and Kahneman (1974) and incorrect belief about the earnings process; the latter assumes that investors follow simple trading strategies conditional on a limited information set. Both cause investors to react slowly news initially and then possibly overreact over the next months, leading to the observed short-term momentum. Fully rational investors are either not present in the market or unable to correct the mispricing due to limits to arbitrage. The problem is expected to be most pronounced among small, illiquid firms.

There is an ongoing debate about whether transaction costs are substantial enough to eliminate momentum profits in practice. Lesmond, Schill and Zhou (2003) estimated that trading costs exceed momentum returns and argued that the strategy is unprofitable due to its selection of small stocks that are especially costly to trade. Korajczyk and Sadka (2006), however, pointed out that the previous conclusion only applies to equally-weighted momentum portfolio that heavily favors small firms. They estimated trading costs using price-impact models and found that value-weighted and liquidity-weighted momentum portfolios remain profitable even after accounting for transaction costs. Momentum among large and liquid firms, it seems, is the more puzzling phenomenon that cannot be easily dismissed with a limit-to-arbitrage argument. In this respect, my explanation for momentum is particularly successful.

There are also attempts at a fully rational framework for momentum. Johnson (2002) proposed one in which a firm's log market value $\log(V)$ is a convex function of a priced state variable p. Then the firm's beta, $d\log V(p)/dp$, is a positive and increasing function of p. An increase in p leads to both an increase in V, or positive recent returns, and higher subsequent returns due to the firm's now higher beta. Sagi and Seasholes (2006) expanded on Johnson's model and interpreted the upper portion of the convex function as risky growth opportunities within firms. Garlappi and Yan (2011) modeled firms as having lower systematic risks as they approach the default boundary, rationalizing the lower portion of the convex function. The link between momentum and financial distress risk, focusing on small firms with large recent losses, has also been explored empirically in Dichev (1998), Campbell, Hilscher and Szilagyi (2008) and Avramov, et al. (2012). My results can be interpreted as being consistent with this literature: for large firms in which financial distress does not come into play, the Fama-French three-factor model is quite capable of explaining momentum profits on its own; for small firms, the addition of a financial distress factor proves sufficient.

Compared to convex value function models, my proposed mechanism for momentum is more flexible because it is not specific to firm equity. Asness, Moskowitz and Pederson (2009) found that the momentum phenomenon exists internationally across many asset classes including bonds, currencies and commodities. In addition, momentum profits are positively correlated across asset classes. These observations are natural implications of my mechanism as long as these different groups of assets share some common risk factors. The same cannot be said about the convex value function models. While they contribute valuable insight into momentum of a particular market, namely the stock market, my explanation is more likely to be the common driving force behind momentum everywhere.

In the next section, I will specify the conditions under which a factor model can generate momentum endogenously, then perform simulation exercises to gauge the magnitude of the momentum alpha that can be produced and explore how various parameters affect it. Sections III will be devoted to empirical results that confirm the intuition and simulation results in the previous section. Section IV will break down the time-varying replicating portfolio to the factor level and illustrate the importance of each component in the alpha-generating and magnification processes. Section V will focus on the unexplained portion of momentum alpha concentrated in small firms and examine the effect of an additional factor on financial distress. Section V will conclude.

2 Theory and Simulation

2.1 Factor Structure and Alpha

I assume an economy whose stock market prices follow a multifactor model such as Ross (1976)'s APT model, i.e., all stock returns obey

$$r_{it} = r_f + \sum_{k=1}^{K} \beta_{ik} f_{kt} + \epsilon_{it}$$

where β_{ik} is the permanent constant beta of asset $i \in \{1, 2, \ldots, I\}$ on the kth systematic factor. f_{kt} is the return of the kth systematic factor at time t and ϵ_{it} is an iid noise term. I will use capital letters to denote vectors and matrices: for example, B_i is the $K \times 1$ vector of β_{ik}'s and F_t is the $K \times 1$ matrix of factor realizations at time t. Then,

$$r_{it} = r_f + F_t' B_i + \epsilon_{it}$$

This is an arbitrage-free world with perfect information and no friction. Any static or dynamic zero-investment trading strategy P using a subset of available stocks will have the following returns

$$r_{Pt} = F_t' W_{Pt} + \epsilon_{Pt}$$

where W_{Pt} is the portfolio's loadings on the factors and ϵ_{pt} a random noise orthogonal to F_t and W_{Pt}. For a static trading strategy where $W_{Pt} = W_P$, a regression of r_{Pt} on the factor returns will yield an alpha of zero. The interesting case is one in which W_{Pt} varies with time. The returns to the dynamic trading strategy can then be broken down into a number of terms:

$$r_t^P = \sum_{k=1}^{K} \tilde{w}_{Pkt} f_{kt} + \sum_{k=1}^{K} \bar{w}_{Pk} f_{kt} + \epsilon_{pt}$$

where

$$\bar{w}_{Pk} = E\left[\bar{w}_{Pkt}\right], \quad \tilde{w}_{Pkt} = w_{Pkt} - E\left[\bar{w}_{Pkt}\right]$$

The alpha of the trading strategy relative to the factors is simply the sum of the alphas generated from each component. Since constant-beta portfolios and the random noise do not produce any alpha, the overall alpha of the strategy is the sum, over k, of the alpha from the following regression:

$$\tilde{w}_{Pkt} f_{kt} = \alpha_k + \beta_k F_t + \epsilon_t$$

Let $\mu = E\left[F\right]$ be the $K \times 1$ vector of expected factor returns and $\Omega = E\left[\left(F-\mu\right)\left(F-\mu\right)'\right]$ be the $K \times K$ variance-covariance matrix of the factors. Then

$$\begin{bmatrix} \hat{\alpha}_k \\ \hat{\beta}_k \end{bmatrix} = \begin{bmatrix} 1 & \mu' \\ \mu & \Omega \end{bmatrix}^{-1} \begin{bmatrix} \mathrm{cov}\left(w_{Pk}, f_k\right) \\ \mathrm{cov}\left(w_{Pk}, f \cdot f_k\right) \end{bmatrix}$$

and

$$\hat{\alpha}_k = \left(1 + \mu'\Omega^{-1}\mu\right) \mathrm{cov}\left(w_{Pk}, f_k\right) - \mu'\Omega^{-1}\mathrm{cov}\left(w_{Pk}, f \cdot f_k\right) \tag{1}$$

where

$$\mathrm{cov}\left(w_{Pk}, f \cdot f_k\right) = \begin{bmatrix} \mathrm{cov}\left(w_{Pk}, f_1 f_k\right) \\ \mathrm{cov}\left(w_{Pk}, f_2 f_k\right) \\ \vdots \\ \mathrm{cov}\left(w_{Pk}, f_K f_k\right) \end{bmatrix}$$

Each α_k is a linear function of the covariances. Of particular importance are $\mathrm{cov}\left(w_{Pk}, f_k\right)$, the covariance between the portfolio's exposure to factor k and the factor return, and $\mathrm{cov}\left(w_{Pk}, f_k^2\right)$, the covariance between the portfolio's exposure to factor k and the factor return volatility. Since factor correlations are relatively stable over time, the other covariance terms contribute negligibly to the portfolio alpha. So

$$\hat{\alpha}_k \approx \left(1 + \mu'\Omega^{-1}\mu\right) \mathrm{cov}\left(w_{Pk}, f_k\right) - \left[\mu'\Omega^{-1}\right]_k \mathrm{cov}\left(w_{Pk}, f_k^2\right) \tag{2}$$

The weights on the two covariance terms depend on the covariance structure of the factors. In the simplest case where all factors are uncorrelated, the above expression is reduced to

$$\begin{aligned} \hat{\alpha}_k &= \left(1 + \sum_{k=1}^{K} \frac{\mu_k^2}{\sigma_k^2}\right) \mathrm{cov}\left(w_{Pk}, f_k\right) - \frac{\mu_k}{\sigma_k^2} \mathrm{cov}\left(w_{Pk}, f_k^2\right) \\ &\approx \mathrm{cov}\left(w_{Pk}, f_k\right) - \frac{\mu_k}{\sigma_k^2} \mathrm{cov}\left(w_{Pk}, f_k^2\right) \end{aligned} \tag{3}$$

The equality comes from the fact that when factors are uncorrelated, $\text{cov}\,(w_{Pk}, f_j f_k) = 0$ for all $j \neq k$. The further simplification comes from the empirically observed fact that μ_k^2/σ_k^2 is usually very small. For each of the Fama-French factors, $\mu_k^2/\sigma_k^2 < 0.02$. Even though the second term in Equation 3 containing $\text{cov}\,(w_{Pk}, f_k^2)$ is supposed to be magnitudes smaller than the first term, the kurtotic nature of the Fama-French factors makes it important as well, and it is helped by the multiplier μ_k/σ_k^2, which is around 2-3 in the data.

The alpha of the overall trading strategy is then

$$\hat{\alpha} \approx \sum_{k=1}^{K} \left[\text{cov}\,(w_{Pk}, f_k) - \frac{\mu_k}{\sigma_k^2} \text{cov}\left(w_{Pk}, f_k^2\right) \right]. \tag{4}$$

Each component of Equation 3 may be small, but when they come together they rise above statistical and economical significance. Momentum is a trading strategy that takes full advantage of all of these components to generate a significant abnormal profit.

Two crucial links bring momentum into the picture. The first is that the beta of the momentum portfolio on a factor is proportional to the realized return of that factor over the period in which past returns are used to rank stocks. When a factor has a positive (negative) realization, stocks with high betas experience a higher (lower) return, keeping everything else constant. High-beta stocks are then more likely to be selected into the winner (loser) part of the momentum portfolio, and the opposite is true for low-beta stocks. The winner-minus-loser position is then proportional to past factor realization, i.e., $w_k \approx M f_{k-}$, where the minus sign in the subscript denotes the previous period (for momentum, it's $t-12$ to $t-2$). It follows that

$$\hat{\alpha} \approx M \sum_{k=1}^{K} \left[\text{cov}\,(f_{k-}, f_k) - \frac{\mu_k}{\sigma_k^2} \text{cov}\left(f_{k-}, f_k^2\right) \right]. \tag{5}$$

The terms within the bracket depend entirely on the factor structure itself, and M acts as a magnifier. The size of M depends on the volatility of the idiosyncratic risk and the dispersion in betas. The former is true because the idiosyncratic component of a firm's return competes with the systematic component in the return sort. It mixes up the ranking and reduces the spread between the betas of winners and losers. The latter is true because momentum selects stocks at the top and bottom of the sort, so high dispersion increases the spread between the betas of winners and losers.

The second link is the autocorrelation structure of factors that fosters a small amount of predictability. In order for momentum to generate a positive alpha, it is helpful if past returns of a factor have predictive power in two ways: first, it predicts positively future returns; second, it predicts negatively future return volatility. I will show in the empirical section below that these two conditions are indeed satisfied, for the most part, for the three

Fama-French factors in the data. It must be noted that not all factors must exhibit these traits; the ones that do contribute to a positive alpha to the momentum portfolio while those that don't offset some of the positive alpha. While not true for the Fama-French factors, it is conceivable that an alternative factor structure can generate a negative momentum alpha.

The two aforementioned links magnifies the small alpha born out of the factor structure and transforms it into a significant one. In theory, momentum can certainly have a positive alpha; the question is whether that alpha is large enough, statistically significant and accounts for the momentum alpha we observe in the data. Through simulation and empirical analysis, I will prove that the answer is, for the most part, yes.

2.2 Simulation

A simulated stock market provides the ideal controlled environment to test the hypothesis that an existing factor structure can generate abnormal momentum returns. The simulation can also predict the amount of momentum alpha that we should observe in the data and enable sensitivity analysis that is not possible to perform empirically.

I will use the CAPM and the Fama-French three-factor model (FF3) as the reference factor structure. I create a stock market with 1,000 stocks ($i = 1, \ldots, 1000$), all starting with a capitalization of $v_{i0} = 1$ and permanent betas on the systematic factor(s) drawn from a distribution. One period is an equivalent of one month in reality. The market operates for 1032 months, which correspond to the period of July 1926 to June 2012 for which FF3 returns are available on Wharton Research Data Services (WRDS)[1]. At time t, each stock $i \in \{1, 2, \ldots, 1000\}$ has a return of

$$r_{it} = \beta_{mi} r_{mt} + \epsilon_{it}$$

assuming CAPM and, alternatively,

$$r_{it} = \beta_{mi} r_{mt} + \beta_{si} r_{st} + \beta_{hi} r_{ht} + \epsilon_{it}$$

with FF3. Subscripts m, s and h stands for MKTRF (market-minus-riskfree), SMB (small-minus-big or size) and HML (high-minus-low or value) factors, respectively. The risk-free rate is assumed to be zero, and ϵ_{it} is drawn from an iid $\mathcal{N}\left(0, \sigma_\epsilon^2\right)$ distribution. The assumption of a zero risk-free rate can also be interpreted as all firms paying out r_f of the firm value as dividend each month, since dividend is not explicitly modeled. The factor returns are taken as the historical factor realizations; therefore, they are identical across different runs of the

[1] URL: https://wrds-web.wharton.upenn.edu/wrds/

simulation. The reason for using actual returns is to leave the factor structure intact while controlling for everything else, so that the effect we see must be originated in the factor structure and nothing else. The capitalization of the firm after time t is

$$v_{i,t} = v_{i,t-1}\left(1 + r_{it}\right)$$

In order to address the possibility that a small portion of the firms may grow disproportionally large and dominating the portfolio weight, a few randomly chosen firms each period have their capitalization set to the median firm size. This procedure in reality does not have a material impact on the results.

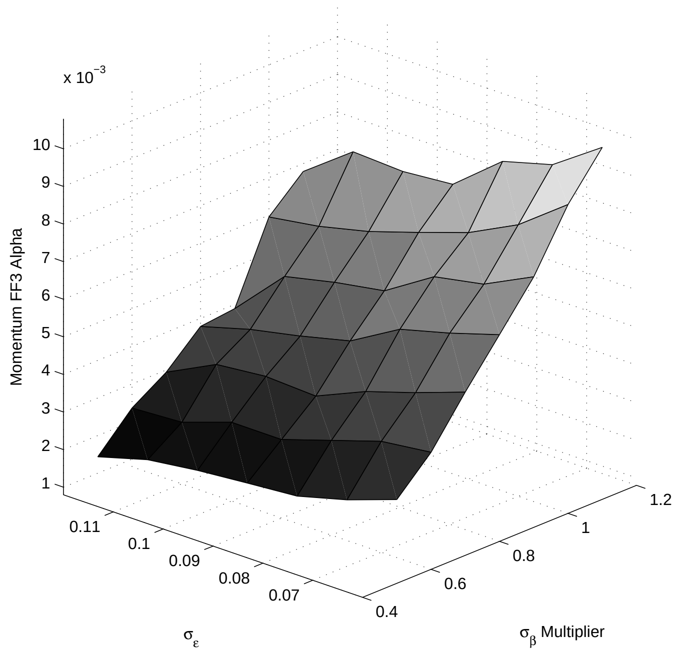
Betas are either drawn from independent normal distributions or an empirical distribution. The latter method is obtained by collecting betas estimated from time-series FF3 regressions of all individual stocks in the CRSP database with a monthly return history of five years or more. The three-factor betas are drawn together to preserve their correlation structure. This joint distribution best describes the long-run average betas of individual firms. The former method takes the standard deviation of all long-run average betas, which are approximately $\{0.5,\ 0.9,\ 0.9\}$ for the three factors, respectively. Then market betas are drawn from $\mathcal{N}\left(1,\ 0.5^2\right)$ while SMB and HML betas are drawn from $\mathcal{N}\left(0,\ 0.9^2\right)$. If we look at the volatility of short-term betas, then they are much higher at $\{0.8,\ 1.3,\ 1.3\}$. The three scenarios, betas from empirical distribution and two different independent normal distributions, are tested to see whether the dispersion in beta matters and how the joint distribution of betas differs from the normal distribution case.

The standard deviation of the idiosyncratic risk, σ_ϵ, is either a constant from the set $\{0.06,\ 0.12\}$ or a time-varying value that equals the value-weighted root mean square error (RMSE) of a two-year rolling-window FF3 regression at each time t. The two constants are chosen to represent the mean RMSE for large and small firms. Since momentum portfolio typically consists of the smallest firms in any group of stocks, the higher idiosyncratic volatility of 12% per month more accurate reflects reality. Since the empirically estimated σ_ϵ varies significantly across time, a third scenario using empirical idiosyncratic risk is also included in the analysis to see whether the time-varying quality affects momentum alpha. During each month, σ_ϵ is taken to be the equally weighted mean root square error of the rolling-window FF3 regression. The average standard deviation is about 10%.

2.3 Simulation Results

Table 1 shows the mean returns, Sharpe ratios and factor regression coefficient estimates for various scenarios in the Fama-French three-factor simulation. Table 2 shows the

Figure 1: FF3 Alpha of the Simulated Momentum Portfolio



Plot of the FF3 alpha of the simulated momentum portfolio as a function of σ_ϵ, monthly volatility of the idiosyncratic component of firm returns and σ_β, the dispersion in the permanent firm betas on the FF3 factors. The y-axis is the multiplier applied to the benchmark σ_β of 0.8 for MKT and 1.3 for SMB and HML. The z-axis is the unconditional FF3 alpha of momentum averaged over 100 runs of the simulated market.

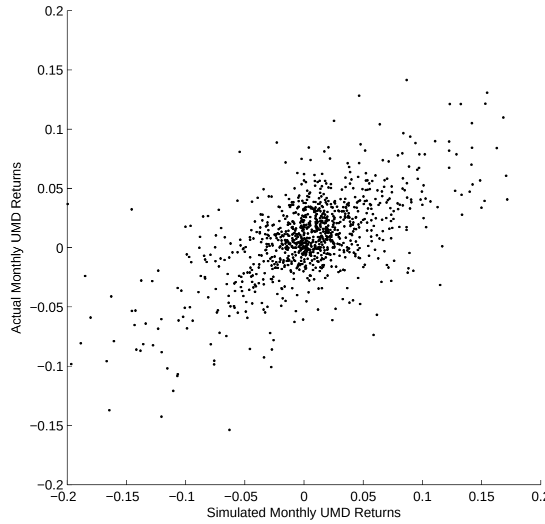
same statistics the CAPM simulation. Each column represents a different parameter pair of betas' joint distribution and idiosyncratic volatility. The statistics are averaged over one hundred runs of the simulation, though the variations between runs are very small so that the statistics are very similar each time. The most striking observation from these tables is that momentum strategies exhibit consistently large and significant profits and alphas across different specifications. For the three-factor model, the alphas range from as low as 0.27% per year to as high as 0.86% per month. Table 2 illustrates that just the market factor itself can already generate a meaningful alpha, about half that seen in the three-factor simulation with comparable parameters. SMB and HML together contribute the other half. However, this does not mean that the market overwhelms the others in a three-factor setting. When multiple factors are present, they compete for the attention of the momentum portfolio. Therefore, each factor is expected to contribute less than it would on its own. In fact, the three factors contribute to the alpha roughly equally in the data.

The amount of momentum profit and alpha depends on a set of familiar parameters. First and foremost is the dispersion in individual firm betas, which determines the degree of magnification that momentum can obtain from the factor structure. Comparing the first three columns and the next three in Table 1, we can see the difference between using high (short-term) and low (long-term) standard deviations of betas. While dispersions in the "high" case is less than twice that in the "low" case, the resulting alpha is more than twice as high for $\sigma_\epsilon = 12\%$. Higher dispersion not only has the benefit of creating a larger spread but also cuts through the noise (idiosyncratic risk) better. Moments other the standard deviation do not seem to matter much, as seen in the comparison between the first and the third groups of columns. Using the long-run empirical joint distribution of beta is similar to only using the standard deviations.

Also important is the volatility of the idiosyncratic risk, as it mixes up betas in the stock return ranking, reducing the spread in beta on which momentum profit is derived. The average level of this volatility seems to be most important, as a 1% increase in monthly volatility reduces momentum alpha by about 0.05% per month or 0.6% per year. The time variation of the volatility, though observed in the data, does not seem to matter. Simulation using the empirical time-varying RMSE looks similar to that using the average of 10%. Figure 1 shows the joint effect of σ_ϵ and σ_β on momentum alpha. The x-axis is the monthly σ_ϵ and the y-axis is the multiplier on the benchmark σ_β of 0.8 for MKT and 1.3 for SMB/HML. The z-axis is the average momentum alpha of the simulation based on the aforementioned parameter set over 100 runs. Other than minor variations, alpha appears to be a linear function of the two parameters, decreasing in σ_ϵ and increasing in σ_β.

Among all the combination of the two parameters above, I will take as benchmark the case where $\sigma_\beta = \{0.8,\ 1.3,\ 1.3\}$ and $\sigma_\epsilon = 12\%$ per month. The reason for former is that momentum is based on one-year returns in the past, which depend on the short-term beta more than the long-term one if betas are time-varying. In this case, there is indeed a significant discrepancy. As I will show in the empirical section below, the long-term estimates of betas are unreliable and lead to a replicating portfolio that has too small an alpha, just like in Table 1. The reason for the latter is also empirical, based on the observation that firms selected into the momentum portfolio, winners or losers, tend to be small firms within the group. Therefore, it is prudent to use a larger idiosyncratic volatility. The benchmark case generates an alpha of 0.57% per month on average, a number that is highly significant both statistically and economically. Figure 2 plots the simulated momentum returns against actual momentum returns. The correlation between the two is quite strong at 68.4% despite the fact that the simulation consists of randomly generated firms with random betas and idiosyncratic risks. Panel A of Table 4 presents statistics of the betas of the simulated momentum portfolio on the FF3 factors. The statistics are averaged over 100 runs of the benchmark simulation. They will be compared to those from the data in the empirical

Figure 2: Simulated vs. Actual Momentum Returns



Scatterplot of actual monthly momentum returns against simulated momentum returns. The correlation between the two return series is 68.4%. The simulated returns are based on a typical run of the benchmark case (σ_β ={0.8, 1.3, 1.3} and $\sigma_\epsilon = 12\%$).

section.

While the magnitude of alphas depends on a few of parameters, the existence of it depends crucially on the factor structure. Table 3 illustrates what happens when the factor structure is altered, fixing the parameters to the benchmark case. If the factor returns are drawn at random from historical values rather than in the order that they appeared (Column 2), then momentum alpha instantly vanishes. This is proof that the factor structure, as it exists in the Fama-French factors, is of paramount importance. On the other hand, factor autocorrelation only matters to a certain point since momentum is a short-term phenomenon. I perform a randomized block bootstrap (Column 3) using the stationary technique by Politis and Romano (1994) to see whether the alpha disappears if returns are randomly arranged by blocks. At an average bootstrap sample size of 24 (2 years), most of the momentum alpha is preserved. This is an indication that the factor structure at the two-year horizon is the leading determinant of momentum profits.

I can push the experiment further and isolate the effect of factor autocorrelation. This is achieved by first drawing factors at random from their historical values and then adding autocorrelation to the random sequence of returns. Both the leverage effect and contemporary correlations between factors are eliminated in this process. Since the actual factor autocorrelations are different between the runs, I run a regression of the form

$$\alpha_j = \gamma_0 + \gamma_1 \rho_j \left(r^{mkt}_{t-12,t-2}, r^{mkt}_t \right) + \gamma_2 \rho_j \left(r^{smb}_{t-12,t-2}, r^{smb}_t \right) + \gamma_3 \rho_j \left(r^{hml}_{t-12,t-2}, r^{hml}_t \right) + \epsilon_j$$

where $j = 1, \ldots, 100$ and ρ_j's are realized correlations. The fit is very tight with $R^2 > 80\%$, meaning that the factor covariances are just shy of being sufficient statistics for the simulated momentum alpha. Column 4 shows the regression results for one run of the simulation where the realized correlations of 9.0%, 11.0% and 4.7% lead to an alpha of 0.39%. Robust estimates of factor correlation in the data are 6.9%, 10.5% and 7.6% for the three factors (more details in the empirical section below), which would imply a momentum alpha of 0.38% per month. It is significantly positive and about two-thirds the 0.57% alpha in the benchmark case. The remainder must be a combination of the leverage effect and other miscellaneous effects.

3 Main Empirical Results

3.1 Data Description

The main data sources are the CRSP (Center for Research in Security Prices) dataset for monthly stock returns and the Fama-French factor data, both of which are available on WRDS (Wharton Research Data Services). The entire CRSP universe of firms is used; it covers the period from January 1925 to December 2011. The Fama-French factors are available from January 1926 to June 2012. To be included in a ranking for portfolio formation, stocks must meet two conditions during the formation month: first, they must be traded on the New York Stock Exchange, the American Stock Exchange or the NASDAQ Stock Market; second, they must have an ending price per share of at least \$1. The latter rule is aimed at eliminating penny stocks suffering extreme liquidity problems and trading frictions. The cutoff does affect the amount of alpha generated from the momentum portfolio, but only slightly and does not materially affect any of the results below. Only very small firms are affected; large firms are defined as those above the NYSE median capitalization, their momentum is virtually unaffected by this rule.

3.2 Variants of the Momentum Portfolio

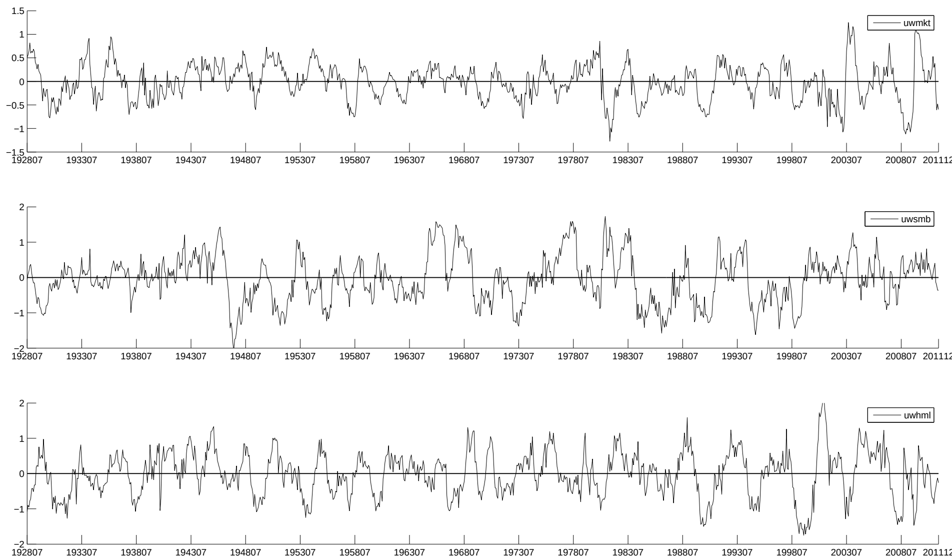
To maintain consistency throughout this paper, I will define the momentum portfolio (also known as UMD or up-minus-down) by ranking stocks based on their cumulative returns in the second to 12th months prior to portfolio formation, i.e., the most recent one-year return excluding the most recent month. This is the canonical definition for momentum and is used in most papers on the subject and the momentum factor data available on WRDS and Ken French's website. The most recent month is excluded due to short-term return reversal, which is a phenomenon distinct from momentum and is the consequence of market microstructure effects according to Jagadeesh and Titman (1995). Table 5 lists the most common variations of the momentum portfolio based on the set of stocks and portfolio weights used in the ranking. It is immediately apparent that large and small firms experience momentum differently, namely that momentum profit and alpha are much higher among small firms. The same difference exists between value-weighted and equally-weighted momentum portfolios based on sorts over the entire sample. Small firms dominate in the equally-weighted portfolio due to their more dispersed returns, while large firms dominate in the value-weighted portfolio due to their size.

3.3 Conditional-Beta Replicating Portfolio

For all of the empirical exercises below, I will assume that the Fama-French three-factor model adequately describes the cross-section of returns for all assets. This is strictly speaking an incorrect assumption, as the FF3 regressions leave large residuals that exhibit both cross-sectional and time-varying features indicative of additional latent systematic factors. However, the omission of latent factors does not invalidate the results below if their influences on pricing is limited. Since the mechanism I have described works for any factor structure, the results should at least be interpreted as a partial picture based on one of the most widely accepted factor structure available. I will argue that the partial picture is actually quite close to the full picture.

The momentum portfolio can be decomposed into two components: the systematic component that loads exclusively on the Fama-French factors and the idiosyncratic component orthogonal to them. The systematic component can in turn be decomposed into three factor components, each of the form $w_t F_t$, where F_t is the return of the factor and w_t is the time-varying weight of the momentum portfolio on that factor. The most intuitive definition of w_t is the weighted average beta of individual stocks on the corresponding factor at the portfolio formation time. For the value-weighted momentum portfolio, the weights are either v_{it} (the "winner" group), 0 (the middle group) and $-v_{it}$ (the "loser" group). I estimate betas of individual stocks from a two-year rolling window, starting at one month prior to the portfolio formation date. The short window offers protection against time variations in betas, in exchange for noisier estimates compared to longer windows. However, since the betas are averaged across hundreds of stocks given the wide top 30% and bottom 30% design of the momentum portfolio, measurement error is significantly attenuated at portfolio level.

Figure 3: Momentum Betas on the Fama-French Factors



Plot of the betas of the momentum portfolio on the FF3 factors through history, computed as the weighted-average betas of individual firms chosen into the momentum portfolio. The weights are equal to the weights momentum assigns to the returns of these firms.

Panel B of Table 4 contains the summary statistics of momentum's loadings on the three factors. The average betas are close to zero and may paint a misleading picture that momentum is weakly correlated with the three factors or is not very volatile. In fact, momentum has high betas on the three factors judging from the high standard deviations of 0.4 to 0.6 and extreme values of magnitude well exceeding 1. However, since the loadings switch frequently from large positive values to large negative values and vice versa, as seen in Figure 3, the mean conceal the high volatility. These rapidly switching betas cause problems in the typical factor regression model because it can only capture the average loadings of a portfolio on a set of factors while ignoring their rich time-varying dynamics. The statistics in Panel B can be compared with those in Panel A from the simulated market. The standard deviation, maximum and minimum values and correlations with factor returns are similar between the two panels, meaning that the simulated market captures well the time series dynamics of momentum. Other moments such as mean, skewness and kurtosis, are somewhat different, but they do not affect alpha, which depends on the comovement between betas and factor returns.

The first three columns of Table 6 show positive and significant alphas between 0.1% to 0.2% per month from pure factor portfolios with time-varying loadings. Altogether, the replicating portfolio generates a Fama-French alpha of over half a percent per month. This figure, as well as a Sharpe ratio of about 10% compares favorably with the value-weighted momentum portfolio or similarly momentum in large firms. When the replicating portfolio is subtracted from the value-weighted momentum portfolio, the resulting residual portfolio has a return of less than 0.1% per month, compared with momentum's 0.5%, and an insignificant FF3 alpha of 0.19%, compared with momentum's highly significant 0.75%. The momentum strategy is no longer profitable after controlling for the replicating portfolio.

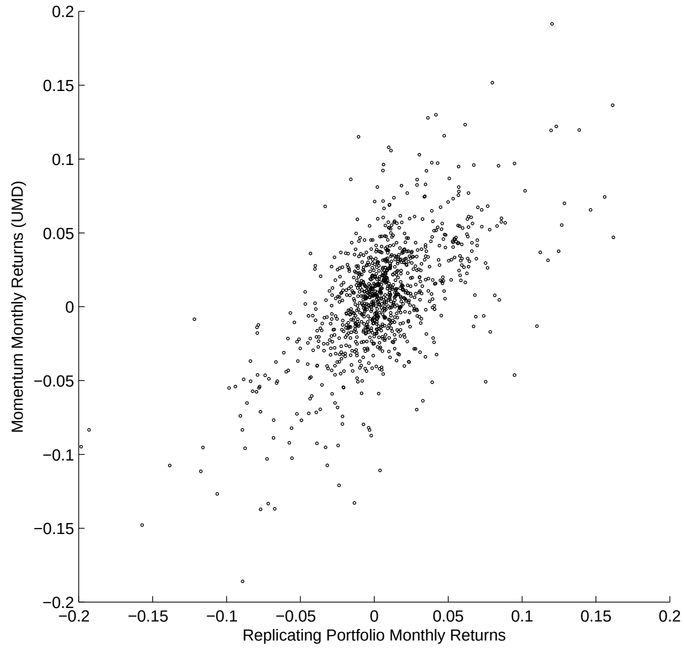
This result is not uniform in the cross-section, however. For firms larger than the NYSE median in terms of capitalization, the story is more or less the same since they dominate in the value-weighted momentum portfolio. The first panel of Table 7 shows that for them, the replicating portfolio explains away most of the abnormal returns. Small firms, on the other hand, have a 30% higher alpha but a 10% lower return volatility. This additional alpha poses a challenge for the three-factor framework because the time-varying factor dynamics of large and small firm momentum portfolios are rather similar. As a result, the replicating portfolio is able to explain less than half of the alpha among small firms. On the other hand, the group of small firms represents a tiny proportion of the market. Their total capitalization is on average only 6% of that of the market, with the remaining space being occupied by large stocks.

When large and small firms are given equal weight, then half of the momentum alpha is explained (Panel 3 of Table 12). An additional observation is that in the 20 years since 1990, the replicating portfolio has performed much better while the momentum portfolio remains roughly the same. The alpha of the residual portfolio is reduced to 0.25% and no longer significant. One may interpret this result as an improvement in the efficiency of the overall market, particularly among small stocks. The residual alpha may be the result of mispricing of small stocks not corrected in the past due to limits to arbitrage. Over time, the market has become more efficient; the residual alpha has shrunk and may continue to shrink in the future.

3.4 Alternative Factor Regressions

A possible critique of the above replicating portfolio is that the post-formation betas of the momentum portfolio are systematically mismeasured. Since the replicating portfolio is a zero-investment trading strategy, it can be arbitrarily scaled to obtain any amount of alpha desired. For instance, if the beta on each factor is overestimated by a factor of $k > 1$, then the replicating portfolio would give an alpha $k\alpha$ greater than the "true" alpha. Then the replicating portfolio would suddenly appear to explain too much of the momentum profits. The converse may also occur where the replicating portfolio with underestimated betas would

Figure 4: Momentum and Replicating Portfolio Returns



Scatterplot of the monthly weighted-average momentum returns and those of the replicating portfolio. The correlation between the two series is 69.5%.

appear to explain too little. Such concern may be alleviated with an alternative factor regression where momentum returns are regressed on returns of each systematic component of the replicating portfolio. In essence, this regression measures the performance of the momentum portfolio relative to not the static systematic factors but the dynamic ones. As seen in Table 8, the alphas from these regressions are roughly equal to the difference between momentum alpha and that of the replicating portfolio. Since the alpha estimates from these alternative regressions do not change when the regressors are scaled by constants, they confirm that there is no systematic mismeasurement of the momentum betas. The coefficient estimates on the three systematic components are different from 1 due to the fact that the factors are correlated and also that the estimated momentum betas are bound to contain noise and different from the "true" betas. In addition, the alpha estimates cannot be realistically obtained, unlike the residual alpha from momentum minus the replicating portfolio, since the betas are fitted from the entire sample and cannot be known at the portfolio formation time.

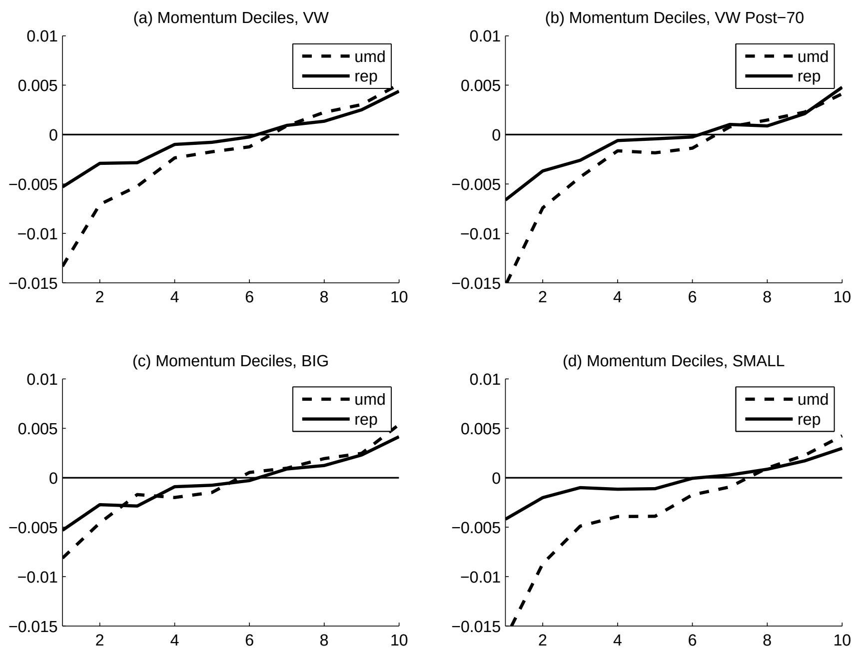
3.5 Decile Portfolios

The cross-section of returns offers a more complete picture of where the conditional beta model succeeds and where it is lacking. Just as the momentum portfolio can be replicated with a time-varying factor portfolio, each decile portfolio based on past return sort can also be replicated in the same manner. The replicating decile portfolios exhibit monotonically increasing alphas from about -0.5% in Decile 1 to zero in Deciles 5 and 6 to about $+0.5\%$ in Decile 10 (Figure 5a). The symmetrical nature of the alpha as a function of the decile contrasts with that for the momentum deciles, which drop much more below zero at Decile 1 than it rises above at Decile 10. At the lower deciles, the replicating portfolios only capture half of the magnitude of momentum's negative alphas, but they fare better at the higher deciles. The discrepancy can mainly be attributed to the small stocks. If the decile portfolios are formed separately for large and small stocks, as shown in Figure 5(c)(d), it is clear that the replicating portfolios perform admirably for large stocks by matching the alpha curve almost perfectly. Accounting for time-varying betas, there is no statistically significant momentum effect for large stocks. The same cannot be said for the small stocks, as the replicating portfolio struggles to keep up at the lower deciles. Figure 5(d) confirms that the highly negative returns of the stocks suffering the worst recent returns present the main challenge to the Fama-French three-factor model.

3.6 Longer Holding Periods

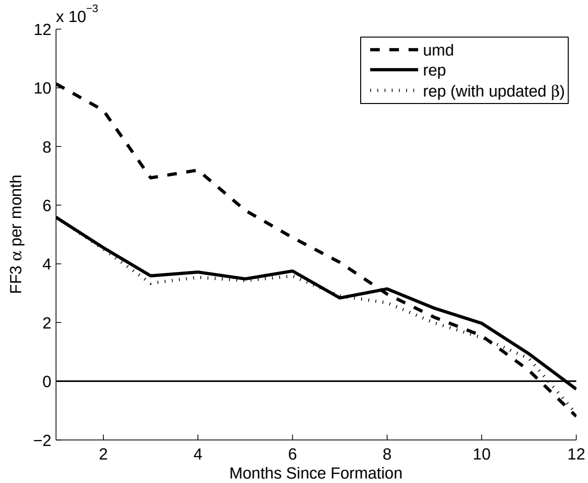
The abnormal return of the momentum portfolio not explained by the replicating portfolio diminishes over the next six months and then disappear altogether. Figure 6 traces the performance of the canonical HH momentum and replicating portfolio relative to the Fama-French factors in the year following formation. The portfolio composition changes slightly over time as a small proportion of stocks are delisted. Their weights are then spread out among the remaining stocks according to the originally weighting scheme. The weights of the replicating portfolio on the factors are computed from individual stock betas estimated during the formation period, which are not revised in the subsequent months. The dotted line in Figure 6 represents the alphas through time of the same replicating portfolio, but with continually updated beta estimates. It looks similar to the original replicating portfolio. In both plots, momentum alpha decreases nearly linearly from about 1% to zero in 12 months, while the replicating portfolio holds steady at around 0.4% for the first half a year, then matches momentum alpha and drops to zero in the later half. Even though the replicating portfolio only explains away half of the momentum FF3 alpha in the first month after formation, it explains about two-thirds of it over the 12 months after formation.

Figure 5: FF3 α of Momentum and Replicating Portfolios by Deciles



Plot of the FF3 alphas of deciles portfolios based on past return sorts, with the alphas of the corresponding replicating portfolios. BIG/SMALL represent subgroups of stocks whose capitalizations are larger/smaller than the NYSE median.

Figure 6: FF3 α of Momentum and Replicating Portfolio over Time



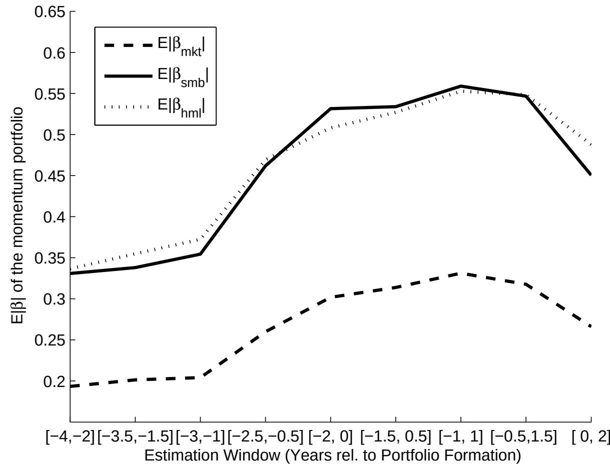
Plot of the FF3 alphas of the canonical (half-half) momentum portfolio and the replicating portfolio over the 12 months after formation. The solid line represents the replicating portfolio based on firms betas estimated using a two-year rolling window prior to the formation month; the dotted line represents the replicating portfolio based on up-to-date firm betas estimated with a two-year rolling window relative to the current month.

Moreover, momentum alpha becomes insignificant within six months after controlling for the replicating portfolio.

4 Alpha Generation

Grundy and Martin (2001) explored the idea of explaining momentum using time-varying exposures to existing systematic factors. They concluded that while factor models can explain a large portion of the variability in momentum returns, they "cannot explain their mean returns." In fact, after subtracting the dynamic replicating portfolio, the alpha of the momentum portfolio increases in most specifications rather than decreases. The author attributed the negative alpha of the replicating portfolio to the negative autocorrelation of the underlying factors. The logic is sound since momentum is a magnified version of a portfolio whose time-varying weight on a factor is a magnified version of the factor's past returns, and positive (negative) autocorrelation of the factor leads to a positive (negative) alpha. The authors, however, have missed several important aspects of the issue, including the effects of sample selection, time-varying stock betas, the leverage effect of factors and the nonlinear relationship between past returns and momentum loadings. They together have a dramatic effect on the dynamic replicating portfolio, namely that it consistently generates a large and positive alpha and explains a substantial proportion of momentum profit; they also lead to a more accurate understanding of how such alpha comes to be.

Figure 7: Beta Estimation Window



Plot of the average magnitude of betas on the three factors, $E|\beta|$, of replicating portfolios based different estimation windows for individual firm betas. The window $[a, b]$ means that firm betas are estimated in monthly FF3 regressions in the period that begins at Year a and ends at Year b relative to the momentum portfolio formation period.

4.1 Time-Varying Betas

Individual stock betas vary over time, and the selection of stocks based on past returns takes this time-varying nature into account. If we assume that the true stock betas are constant over time, then the window during which stocks betas are estimated prior to portfolio formation should not systematically bias the betas of a portfolio consisting of many stocks. Figure 7 is a clear illustration to the contrary: it plots the average magnitude of the momentum portfolio's betas on the three factors computed from individual stock betas estimated at different times. There are nine different scenarios, each of which is a two-year estimation window starting at six months apart. The middle data points represent betas estimated from the most recent two years. There is a noticeable rise of over 50% in the magnitude of these betas from a year before formation to the formation month. Individual stock betas are clearly not constant over time.

If stock betas change over time, then the momentum portfolio can take advantage of this feature. Since stocks are ranked based on past year's returns which are products of past factor returns and betas during the past year, those with highly positive or negative realized betas, not long-run average betas, are chosen into the momentum portfolio. Whatever the time-series dynamics of the individual stocks, those that have recently experienced a shock that made its betas more positive or more negative are more likely to be winners or losers. Therefore, the magnitude of momentum's betas on the three factors should be much higher than what are implied by long-run betas of individual stocks. According to

Figure 7, it is higher by between 50% and 60%. This large difference adds a new dimension to the momentum portfolio and has two important implications. First, as long as there is time variation in stock betas, even if it is only i.i.d., momentum weights are more volatile and abnormal profits magnified. Changing stock betas is not a controversial claim: many previous studies including Harvey (1989), Ferson and Harvey (1991, 1993) and Ferson and Korajczrk (1995) offer evidence of such. Regardless of the exact specification, time-varying betas magnify the momentum portfolio by offering a wider range of realized betas at any given time. Momentum always seizes the opportunities and pick stocks with the more extreme realized betas.

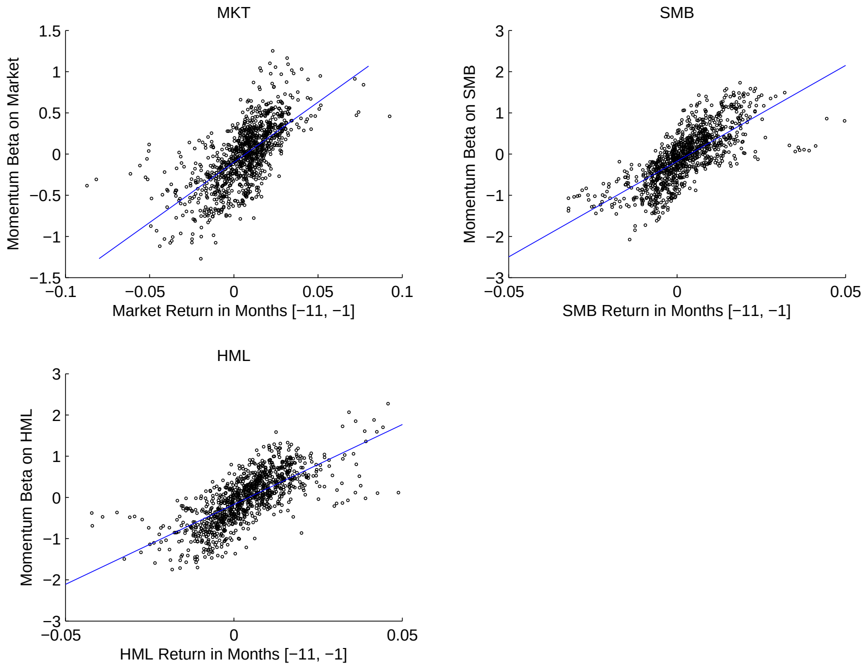
The second implication is that long run betas of individual stocks are bound to lead to poor replicating portfolios; short window must be used in this case. Table 12 compares the performance of replicating portfolios based on different estimation windows. The 2-4 year window and the five-year window (used by Grundy and Martin, 2001) are similar in that they both give estimates of long-term average betas; they both produce replicating portfolios whose alphas are too small. The recent two-year window, as expected, takes into account the recent changes in stock betas picked up by the momentum portfolio, and produces an alpha that is significantly higher.

There is also evidence that the changes in stock betas are persistent over time. In Figure 7, the average magnitude of the momentum betas increases by over 50% in the two years prior to portfolio formation but decreases by less than 20% in the two years that follow, still significantly higher than the level at the previous average level. In fact, a replicating portfolio constructed using post-formation betas, while clearly not a tradable strategy, yields a Fama-French alpha of about 0.5%. This result refutes the possible criticism that the recent shocks to stock betas are temporary and that those betas revert back to the original level immediately after the formation period; instead, it suggests that such shocks are very persistent so that the short-window estimation is the correct method.

4.2 Autocorrelation and the Leverage Effect

The root cause of abnormal return in the replicating portfolio is the factor structure itself. Two effects are at work to generate a positive alpha: the slightly positive autocorrelation and the leverage effect. Since factor autocorrelation is weak, it is susceptible to outliers influencing the coefficient estimate, particularly when the factor is highly kurtotic. The first look can be deceiving: the market factor and the HML factor have autocorrelation coefficients of 1.5% and 0.9%, respectively, and both are insignificant. SMB has a significant coefficient of 6.5%. They seem to contradict the fact that each factor component of the replicating

Figure 8: Scatterplots of Momentum Betas and Past Factor Returns



Scatterplots of momentum betas against past factor returns with OLS slope estimates, showing the downward bias caused by outliers.

portfolio generates a significantly positive alpha. However, these low estimates are due to outliers. Removing around 5% of the most extreme realizations of past factor returns changes the estimates dramatically. Market, SMB and HML now have autocorrelation coefficients of 6.9%, 10.5% and 7.6%, respectively. All are significant at 1%. These extreme realizations of factor returns, while able to skew the autocorrelation estimates, turn out to have little impact on the replicating portfolio. The reason is that as the realized factor return becomes larger in magnitude, its ability to overcome idiosyncratic returns and align winners and losers with betas increases. After a certain point, however, the marginal impact diminishes and eventually disappears since winners and losers are already well aligned with betas. Momentum's loading on a factor is roughly a linear function of the past factor return in the majority of the periods when the return is close to zero. In periods with extremely large factor realizations, the actual loading is much smaller in magnitude than that linear function would suggest. Figure 8 illustrates the problem where a few outliers reduce significantly the slope of the fitted line through the momentum betas and factor returns scatterplot. Robust regressions using Huber weights that reduce the influence of outliers lead to higher estimates.

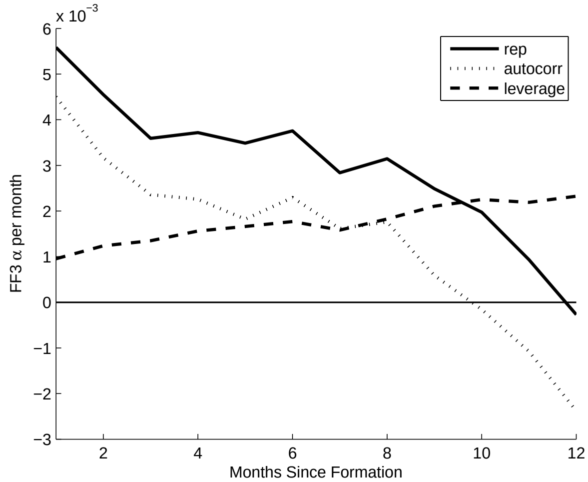
Even though the relationship between factor returns and momentum betas is not linear, a linear function is adequate because the vast majority of factor realizations are close to the origin. A simple linear regression between the two variables yields an R^2 of around $50-60\%$. More elaborate methods, including nonlinear ones proposed by Grundy and Martin (2001), do not improve on the predictive power. GM assumed normal distributions for all returns as well invariant distribution of betas and idiosyncratic component of returns; none of these are true in the data. A linear relationship allows the dissection of the momentum loading on a factor w, into the past factor returns $F_{t-12,t-1}$ times a multiplier M. The exact mechanism by which the features of the factor structure lead to momentum alpha is revealed.

A solution that is ad hoc but effective is to remove the few outliers and look at the "core" factor structure and relationship between past returns and momentum betas. The two panels of Table 13 show the stark contrast between the full sample and truncated sample statistics. Periods in which the absolute values of factor returns exceeding a cutoff point are omitted in the truncated analysis. For the three factors they are 0.04, 0.03 and 0.03, respectively. Altogether only 5% of the sample is omitted, but the difference it makes is very large. The autocorrelation coefficients become significant for market and HML and more significant for SMB. The "autoregressive portfolios", formed by factors scaled with their own past returns, have insignificant alphas for SMB and HML, which is at odds with the fact that the corresponding components in the replicating portfolio have significant alphas. After the outliers are removed, however, Both of these autoregressive portfolios generate positive and significant alphas.

Table 14 gives a detailed account of momentum alpha down to the factor structure level, made possible with the analysis of the core sample without outliers. The alpha of the autoregressive portfolio, $F_{t-12,t-2}F_t$, is mainly the sum of the covariance between $F_{t-12,t-2}$ and F_t and the covariance between $F_{t-12,t-1}$ and F_t^2. The first is the factor autocorrelation and the second the leverage effect. All three factors have significant core autocorrelation, while the market has a significant leverage component accounting for 36% of its autoregressive portfolio alpha. The alphas of the autocorrelation portfolio, though small on the order of 10^{-5}, become magnified with the large multiplier M, which is estimated by regressing momentum beta on the past returns of the corresponding factor. The result is a significantly positive alpha around 0.15% for each component of the replicating portfolio. In total, factor autocorrelation and the leverage effect contribute an alpha of 0.48% per month, which is more than 85% of the total amount in the replicating portfolio. The remainder is a combination of several minor effects including the slight predictability of factor covariance, the nonlinearity of the momentum betas as a function of past factor returns and the predictability of the residual portion.

It would appear, upon first glance, that the leverage effect is very small, only a fifth of the autocorrelation effect. This is the case mainly because market is the only factor with

Figure 9: Contribution of the Autocorrelation and Leverage Effects



Plot of FF3 alpha of momentum broken down into contributions from return predictability and volatility predictability, according to Eq. 4, over a period of one year after portfolio formation.

significant leverage effect. Over the next 12 months, however, the situation changes. Figure 9 plots the breakdown of momentum alpha into the two effects, according to Eq. 4, over a period of one year . While factor autocorrelation diminishes quickly down to zero and even into negative territory, the leverage effect holds steady and actually increases its contribution to the portfolio alpha to over 0.2% per month. In the year following formation, the average alpha generated from the leverage effect is about the same as that from autocorrelation.

5 Residual Alpha

About 40% of the momentum alpha that is not accounted for by the replicating portfolio is concentrated among small, losing stocks. They represent firms that are already small but have suffered a large losses recently; these are firms on the verge of being delisted and possibly going into bankruptcy. Taking advantage of this profit opportunity is difficult and costly, as one must take a short position in these very small stocks and rebalance the portfolio every month. The replicating portfolio has already removed what would otherwise appear to be easier and cheaper arbitrage opportunity, i.e., to take a long position in recent winners among large firms and expect an abnormal positive profit. The remaining portion, for whatever reason that exists, presents a high barrier to entry for potential arbitrageurs. Regardless of the limits to arbitrage argument, however, the question remains of why these small losing firms suffer substantially lower-than-expected returns.

5.1 Calm and Turbulent Periods

One possibility is that the Fama-French model is misspecified, and there exist important latent systematic risk factors that generate positive alpha among small firms. These latent factors would be hiding in the idiosyncratic risk portion of the three-factor regression. If the Fama-French model is assumed to be the correct pricing model, then the momentum portfolio should have high alpha during times when past factor returns are large in either direction and low otherwise. According to Table 9, this is true for the replicating portfolio but not for the momentum. If we divide the last 80-some years into two halves based on the absolute value of market returns in the one year prior to portfolio formation, we can see that the replicating portfolio generates an alpha of 0.7% per month conditional upon high market action and 0.4% per month otherwise. In the meantime, the momentum portfolio generates an alpha of 0.8% per month conditional upon high market action but 1.0% per month otherwise. The results are similar when conditioning on the realizations of all three factors jointly (Panel 4). Here a turbulent period is one in which the realized factor return of at least one of the factors lies outside of the 10th-90th percentile range of historical returns for that factor. As in the previous case with the market factor, the replicating portfolio performs admirably during turbulent times but is less effective during calm times. This striking difference shows when the replicating portfolio succeeds and when it fails: it functions as expected and kicks into high gear whenever there is a significant movement in one of the factors. This factor becomes the dominant one in the winner and loser selection, and subsequent momentum returns are strongly tied to its subsequent returns. When there is little movement in all of the factors, winners and losers are aligned along the "idiosyncratic" portion of their returns, which likely contains latent factors. The influence of these latent factors is quite strong, creating a return spread of about 0.6% per month on top of what the replicating portfolio can provide.

There is also the possibility that the return spread during calm periods is caused by behavioral biases. Since the major systematic risk factors have barely moved in the last 12 months, investors have difficulties judging the relative performance of stocks. The regression of individual stock returns on the three factors yields very noisy estimates because of the low variation in the regressors, so investors must focus on other aspects of the firm. Therefore, the recent returns of individual stocks become more important, as investors put more weight on recent news. Then behavioral biases such as under-reaction to news are likely exacerbated, leading to a significant alpha not accounted for by the systematic risks.

5.2 Financial Distress Factor

Financial distress risk is often linked to momentum in the literature and has been rationalized as a systematic risk in Garlappi and Yan (2001). They showed that when stocks are grouped by different levels of default probabilities, the momentum portfolio produces significantly positive Fama-French alphas in all groups but higher alphas in groups with higher default risk. Since distressed firms are most likely to be small ones who have recently experienced highly negative returns, it is possible that financial distress risk may indeed explain some or all of the residual alpha.

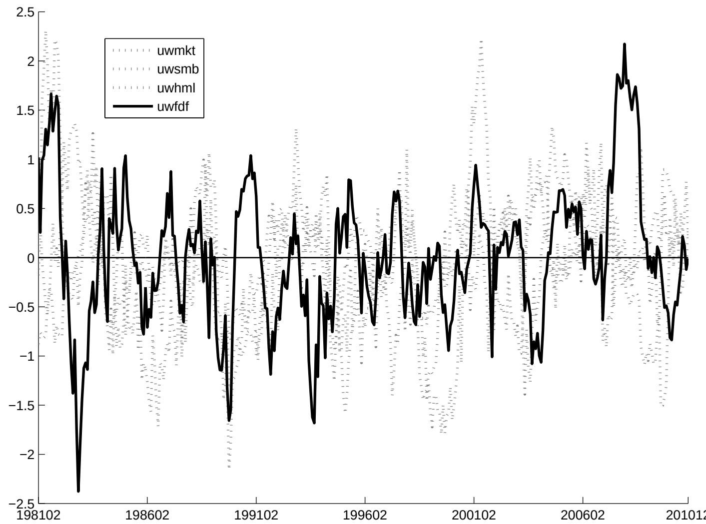
To construct a financial distress factor (FDF), I first estimate each individual stock's default probability in a fashion similar to Campbell, Hilscher and Szilagyi (2010). At the end of each year, I perform a panel logistic regression of the form (Campbell, et al. 2010 Eq. 1)

$$P_t\left(Y_{i,t+1}=1\right)=\frac{1}{1+\exp\left(-\alpha-\beta x_{i,t}\right)}$$

using all available COMPUSTAT firm data prior to that point. $x_{i,t}$ is a vector of firm characters: net income to total asset ratio (NITA), total liabilities to total asset ratio (TLTA), log excess return relative to the S&P 500 index in the most recent month (EXRET), standard deviation of returns in the past three months (SIGMA), relative size of the firm relative to the S&P 500 index (RSIZE), cash and short-term investments over the market value of total assets (CASHMTA), market-to-book ratio (MB) and log price per share winsorized above \$15 (PRICE). All accounting indicators are computed from the most recent quarterly report. $Y_{i,t+1}$ is an indicator that takes a value of 1 if the firm defaults in the next 12 months with a delisting code of 4XX (liquidations) and 5XX (dropped or stopped trading). The most recent year is excluded because the firms' future prospects are not known. The coefficient estimates are then applied to each firm to predict the likelihood that it will be delisted in the next year.

At the end of each month, all stocks are assigned and then sorted by distress probabilities implied by the most recent logistic regression results and their current firm characteristics. A portfolio is formed by taking a long position in the firms with the 30% lowest distress probabilities and a short position in those with the 30% highest distress probabilities. The return on the financial distress factor is the return of the aforementioned portfolio in the subsequent month. Since COMPUSTAT only has coverage starting in the 1970s, and the number of firms available is spotty until the late 70s, the financial distress factor is available from 1979 to 2010. During this period, the canonical "half-half" momentum portfolio produces a FF3 alpha of 0.86% per month. Controlling for the distress factor in a four-factor regression, momentum still yields a significant alpha of 0.63% per month (Table 15). As

Figure 10: Momentum Beta on the Financial Distress Factor



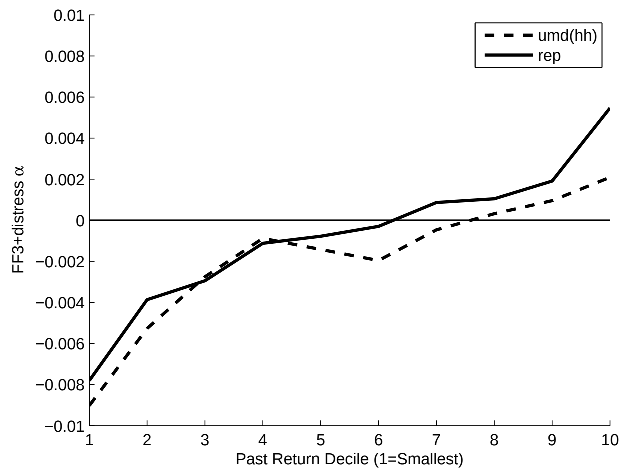
Plot of the betas of the momentum portfolio on the financial distress factor (and the FF3 factors) from 1980 to 2010.

expected, momentum has a loading of 0.44 on the distress factor, whose positive expected return helps to reduce its alpha. However, it is important to note that the unconditioanl Fama-French model with the distress factor added to it does not explain all of momentum's abnormal profits; for that, the dynamic replicating portfolio is necessary.

The replicating portfolio retains an alpha of 0.57% after controlling for the additional factor. The alpha has hardly changed because the replicating portfolio is simply a weighted factor portfolio and unrelated to distress. Momentum and its replicating portfolio now generate almost the same amount of alpha, and the difference is tiny and insignificant. Distress appears to be the sole source of the residual alpha. A decile plot (Figure 11) shows exactly what has changed when the fourth factor is added. In the lower deciles, momentum loads negatively on the distress factor, so the abnormal return is much less negative than before. The replicating portfolio can now match the left tail. In the higher deciles, the replicating portfolio outperforms momentum now because momentum loads negatively on the distress factor there. The difference is not statistically significant, however.

I am agnostic about whether the distress risk factor arises due to behavioral biases or compensation for risks. Garlappi and Yan (2011) would argue that firms take on less systematic risk as they approach default, so the lower returns of distressed firms are justified in terms of risk exposures. On the other hand, a behavioral argument in which investors flee from failing firms due to disastrous recent performances, causing fire sale and contributing to further price decline, may also justify this observed effect. It would be very difficult for

Figure 11: FF3 + FDF α of Momentum and Replicating Portfolio by Deciles



Plot of the return deciles on which the canonical (half-half) momentum portfolio is based, with the corresponding returns of the replicating portfolio.

arbitrageurs to take advantage of this apparent arbitrage opportunity. Regardless of what distress risk is, this section highlights the fact that the dynamic replicating portfolio based on the Fama-French factors is distinct from it, and the two of them together are just enough to explain all of momentum profits. The implications are twofold. First, the breakdown between the mechanical effect from the Fama-French factor structure and the financial distress effect is clear. Second, the sum of the two represents the totality of momentum's abnormal returns, eliminating the need for other mechanisms and risk factors.

6 Conclusion

I have shown through simulation and empirical tests that a multifactor asset pricing model is capable of explaining a large portion of momentum profits without resorting to behavioral biases and additional latent systematic risks. Two features inherent in factor structures, positive autocorrelation and the leverage effect, allow for the creation of small, positive alphas in factor portfolios where the weights are equal to past returns. Momentum loads selectively on factors depending on their realized returns and magnifies alphas by selecting stocks with highly positive and negative betas in a long-short portfolio. Momentum's weights on each factor is roughly the product of the past return of the factor and the spread of individual stock betas on that factor in the cross-section. The former provides two sources of positive alpha, while the latter provides the magnification. The time-varying nature of individual stock betas is very important, as momentum gains additional magnification power by selecting stocks who have experienced large shocks to their betas recently, not ones with higher (or more negative) average betas.

During the first month after portfolio formation, the replicating portfolio based on time-varying loadings on the factors is capable of explaining half of canonical momentum's Fama-French alpha. For the value-weighted momentum portfolio, however, the replicating portfolio explains 75% of its FF3 alpha, with the remaining portion statistically insignificant. Explaining the value-weighted momentum is arguably more important, since previous literature has shown that such portfolio, dominated by large stocks, is still profitable after accounting for transaction costs, whereas the equally-weighted portfolio representing small stocks is not. During the year after formation, the replicating portfolio explains an increasingly large portion of momentum's abnormal profit until it reaches 100% in Month 8. The role of the leverage effect is small in the beginning relative to the large alpha generated by factor autocorrelation. However, the latter is short-lived, and the leverage effect becomes stronger and dominates the autocorrelation effect six months from formation. The two sources contribute equally to the abnormal return of the replicating portfolio over time.

The remaining alpha not explained by the replicating portfolio can be attributed to the underperformance of very small firms with recent losing streaks. Financial distress risk appears to be the sole factor at work: a distress factor based on firms' predicted failure rate can explain away the remaining alpha completely. The four-factor model demonstrates the power of the conditional replicating portfolio: it is capable of explaining away the entirety of the momentum returns, whereas an unconditional four-factor regression would yield a highly significant alpha and suggest that the model is still inadequate.

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Tables

Table 1: Simulated Momemtum Based on the Fama-French Three-Factor Model

A simulated stock market with 1,000 stocks is operated for 1,032 months, corresponding to July 1926 to June 2012 in history. Each stock starts with a capitalization of 1 and is assigned a permanent set of three Fama-French betas drawn from either independent normal distributions with means $\{1, 0, 0\}$ and standard deviations $\{0.5, 0.9, 0.9\}$ (low std) or$\{0.8, 1.3, 1.3\}$ (high std) for mkt, smb and hml factors, respectively, or from the empirical joint distribution of long-term betas from all stocks in CRSP. Returns follow the Fama-French three-factor model with i.i.d. idiosyncratic risk drawn from the distribution $\mathcal{N}\left(0, \sigma_\epsilon^2\right)$, where σ_ϵ is either a constant or the average RMSE in the FF3 regressions of all stocks during that period in history. The factor returns are taken from and fixed at the realized historical values. Dividend yield is equal to the risk-free rate. At the end of each month, stocks are sorted into deciles by their cumulative returns in the past 12 months (omitting the most recent month). The momentum portfolio is formed by taking a value-weighted long position in the top three deciles and a value-weighted short position in the bottom three deciles. The simulated market is repeated 100 times, and all statistics are averages across simulations.

β	independent normal (low std)			independent normal (high std)			long-run empirical		
σ_ϵ	6%	EM	12%	6%	EM	12%	6%	EM	12%
$\bar{r}$	0.0036**	0.0023*	0.0018	0.0069***	0.0049**	0.0042*	0.0034**	0.0024*	0.0018
	[0.0016]	[0.0013]	[0.0012]	[0.0029]	[0.0025]	[0.0023]	[0.0017]	[0.0014]	[0.0013]
$\bar{r}/\sigma$	0.0639	0.053	0.0477	0.0787	0.0691	0.0595	0.0682	0.0444	0.0451
FF3 α	0.0046***	0.0032**	0.0027**	0.0086***	0.0065***	0.0057***	0.0051***	0.0038***	0.0032***
	[0.0015]	[0.0012]	[0.0011]	[0.0026]	[0.0022]	[0.0020]	[0.0016]	[0.0013]	[0.0011]
β_{mkt}	-0.25**	-0.20**	-0.19**	-0.41**	-0.35**	-0.35**	-0.29***	-0.23***	-0.23***
	[0.1]	[0.08]	[0.08]	[0.17]	[0.15]	[0.14]	[0.10]	[0.09]	[0.08]
β_{smb}	0.39**	0.26**	0.28**	0.61**	0.47**	0.50**	0.28*	0.19	0.22*
	[0.16]	[0.12]	[0.12]	[0.26]	[0.21]	[0.21]	[0.15]	[0.12]	[0.11]
β_{hml}	-0.12	-0.09	-0.10	-0.19	-0.17	-0.18	-0.16	-0.12	-0.13
	[0.19]	[0.16]	[0.16]	[0.30]	[0.27]	[0.27]	[0.19]	[0.16]	[0.16]
Avg R^2	9.9%	8.5%	10.3%	9.1%	8.7%	10.5%	10.5%	9.8%	11.6%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 2: Simulated Momemtum Based on the CAPM

A simulated stock market with 1,000 stocks is operated for 1,032 months, corresponding to July 1926 to June 2012 in history. Each stock starts with a capitalization of 1 and is assigned a permanent set of market betas drawn from either independent normal distributions with mean of 1 and standard deviation of 0.5 (low std) or 0.8 (high std), or from the empirical distribution of long-term market beta from all stocks in CRSP. Returns follow the CAPM with i.i.d. idiosyncratic risk drawn from the distribution $\mathcal{N}\left(0, \sigma_\epsilon^2\right)$, where σ_ϵ is either a constant or the average RMSE in the CAPM regressions of all stocks during that period in history. The market returns are taken from and fixed at the realized historical values. Dividend yield is equal to the risk-free rate. At the end of each month, stocks are sorted into deciles by their cumulative returns in the past 12 months (omitting the most recent month). The momentum portfolio is formed by taking a value-weighted long position in the top three deciles and a value-weighted short position in the bottom three deciles. The simulated market is repeated 100 times, and all statistics are averages across simulations.

β	independent normal (low std)			independent normal (high std)			long-run empirical		
σ_ϵ	6%	EM	12%	6%	EM	12%	6%	EM	12%
$\bar{r}$	0.0013	0.0009	0.0007	0.0030	0.0023	0.0017	0.0016	0.0010	0.0007
	[0.0010]	[0.0008]	[0.0007]	[0.0019]	[0.0016]	[0.0014]	[0.0011]	[0.0009]	[0.0008]
$\bar{r}/\sigma$	0.0360	0.0208	0.0435	0.0562	0.0404	0.0449	0.0418	0.0376	0.0209
FF3 α	0.0022**	0.0016**	0.0015*	0.0046**	0.0037**	0.0032**	0.0027**	0.0020**	0.0016*
	[0.0010]	[0.0008]	[0.0007]	[0.0020]	[0.0016]	[0.0014]	[0.0012]	[0.0009]	[0.0008]
β_{mkt}	-0.15	-0.13	-0.12*	-0.26	-0.24	-0.23	-0.18	-0.15*	-0.14*
	[0.09]	[0.08]	[0.07]	[0.17]	[0.15]	[0.14]	[0.11]	[0.09]	[0.08]
Avg R^2	7.5%	7.9%	8.2%	6.0%	7.1%	8.2%	7.5%	8.5%	8.7%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 3: Simulation with Randomized Fama-French Factors

The benchmark simulation setup is one with $T = 1032$, betas drawn from independent normal distributions with high standard deviations $\{0.8, 1.3, 1.3\}$ and i.i.d. idiosyncratic risk drawn from the distribution $\mathcal{N}(0, 0.12)$. All other scenarios have this setup and $T = 3000$ plus the following difference. The randomized factor scenario uses factor returns drawn individually from the historical distributions of factors without replacement. The block bootstrap scenario uses factor returns drawn in random length blocks (average length = 24, or 2 years) without replacement. The artificial autocorrelation scenario uses factor returns drawn individually from the historical distributions of factors and then injected with positive autocorrelation. The sample run has realized autocorrelations of 9.0% , 11.0% and 4.7% on the three factors. The next column regresses the observed momentum alpha on the realized autocorrelations across different runs of this scenario. The empirical core autocorrelations yield a predicted alpha of 0.38% per month given these estimates.

Case	Benchmark	Randomized Factors	Block Bootstrap	Artificial Autocorrelation		
				Sample Run	Regression across 100 Runs	
$\bar{r}$	0.0042*	0.0016	0.0031**	0.0052***		
	[0.0023]	[0.0016]	[0.0012]	[0.0006]		
$\bar{r}/\sigma$	0.0595	0.0148	0.0765	0.1227	LHS = UMD FF3 α	
FF3 α	0.0057***	0.0002	0.0047***	0.0039***	Intercept	0.00024
	[0.0020]	[0.0015]	[0.0012]	[0.0006]		[0.00038]
β_{mkt}	-0.35**	0.00	-0.29***	0.07**	$\text{cov}\left(r^{mkt}_{t-12,t-2} r^{mkt}_t\right)$	0.0190***
	[0.14]	[0.07]	[0.08]	[0.03]		[0.0038]
β_{smb}	0.50**	0.25	0.35*	0.17	$\text{cov}\left(r^{smb}_{t-12,t-2} r^{smb}_t\right)$	0.0109***
	[0.21]	[0.17]	[0.12]	[0.11]		[0.0039]
β_{hml}	-0.18	0.20*	-0.17	0.14***	$\text{cov}\left(r^{hml}_{t-12,t-2} r^{hml}_t\right)$	0.0144***
	[0.27]	[0.11]	[0.15]	[0.04]		[0.0039]
Avg R^2	10.5%	4.5%	8.9%	3.6%	Adj R^2	80.8%

[ ]: Newey-West standard errors with 6 lags (except OLS in Row 6); */**/***: statistically significant at 10/5/1%

Table 4: Summary Statistics of Momentum Betas

Panel A contain statistics of the momentum portfolio betas (w) on the FF3 factors in the simulated market with the benchmark parameter values, averaged over 100 runs. Panel B contain the same statistics of the momentum portfolio betas estimated from the data. At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The portfolio betas (w) are the value-weighted average betas of the individual stocks chosen into the momentum portfolio.

A. Simulations (Benchmark Case)

	Univariate Statistics							Correlations				
	Mean	Median	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum	$MKT_{t-12,t-2}$	$SMB_{t-12,t-2}$	$HML_{t-12,t-2}$	w_{SMB}	w_{HML}
w_{MKT}	0.1079	0.1624	0.5432	-0.3845	-0.0672	-1.5070	1.4758	**78.32%**	21.43%	9.84%	15.50%	12.58%
w_{SMB}	0.0970	0.1247	0.7795	-0.1350	-0.1463	-2.1616	2.2189	21.43%	**79.47%**	5.50%		2.82%
w_{HML}	0.2161	0.2404	0.8759	-0.0853	-0.3876	-2.2934	2.5062	6.57%	-3.56%	**78.40%**		

B. Data

	Univariate Statistics							Correlations				
	Mean	Median	Std. Dev.	Skewness	Kurtosis	Minimum	Maximum	$MKT_{t-12,t-2}$	$SMB_{t-12,t-2}$	$HML_{t-12,t-2}$	w_{SMB}	w_{HML}
w_{MKT}	-0.0017	0.0154	0.3692	-0.0641	0.3378	-1.2980	1.2147	**66.18%**	15.16%	14.02%	9.12%	14.74%
w_{SMB}	-0.0542	-0.0548	0.6444	0.0899	-0.1975	-2.0127	1.7449	10.83%	**72.23%**	4.93%		5.31%
w_{HML}	-0.0147	0.0327	0.6120	-0.0517	-0.0055	-1.6924	2.2288	11.56%	8.03%	**72.31%**		

Table 5: Variants of Momentum

At the end of each month, stocks are sorted into deciles by their cumulative returns in the past 12 months omitting the most recent month. The momentum portfolio is formed by taking a long position in the top three deciles and a short position in the bottom three deciles; it is then held for one month forward. Each column is a different method of weighting stocks in the long and short portions of the portfolio: VW - value-weighted; EW - equally weighted. B/S/HH - stocks are divided into two groups based on whether their capitalizations are larger or smaller than the NYSE median that month; a value-weighted momentum portfolio is formed for each group, B as the large cap group and S as the small cap group. HH = (B+S)/2.

Weights	VW	EW	HH	B	S
$\bar{r}$	0.0049***	0.0085***	0.0065***	0.0046***	0.0085***
	[0.0015]	[0.0014]	[0.0014]	[0.0015]	[0.0015]
$\bar{r}/\sigma$	0.1044	0.1798	0.1428	0.0955	0.1723
FF3 α	0.0075***	0.0117***	0.0094***	0.0071***	0.0118***
	[0.0013]	[0.0012]	[0.0012]	[0.0013]	[0.0013]
β_{mkt}	-0.16**	-0.22***	-0.20***	-0.15**	-0.26***
	[0.07]	[0.06]	[0.06]	[0.07]	[0.06]
β_{smb}	0.07	-0.11	-0.02	0.07	-0.24
	[0.09]	[0.09]	[0.08]	[0.09]	[0.06]
β_{hml}	-0.44***	-0.44***	-0.42***	-0.45***	-0.39***
	[0.12]	[0.13]	[0.12]	[0.12]	[0.13]
Adj. R^2	16.4%	23.29%	20.01%	16.10%	20.79%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 6: Replicating Portfolio with Time-Varying Beta

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio, REP(Factor), is the factor itself scaled by momentum's beta on that factor. The replicating portfolio (REP) is the sum of the three components. The residual portfolio (RES) is the difference between the value-weighted momentum portfolio (UMD-VW) and the replicating portfolio (REP).

	REP(MKT)	REP(SMB)	REP(HML)	REP	RES	UMD-VW
$\bar{r}$	0.0007	0.002***	0.0014*	0.0041***	0.0008	0.0049***
	[0.0008]	[0.0007]	[0.0008]	[0.0014]	[0.0012]	[0.0015]
$\bar{r}/\sigma$	0.0306	0.1041	0.0596	0.0964	0.0234	0.1044
info ratio	0.0350	0.1042	0.0931	0.1382	0.0551	0.1726
FF3 α	0.0015**	0.0020***	0.0021***	0.0056***	0.0019	0.0075***
	[0.0008]	[0.0006]	[0.0007]	[0.0012]	[0.0012]	[0.0013]
β_{mkt}	-0.12***	-0.01	-0.09**	-0.21***	0.05	-0.16**
	[0.04]	[0.01]	[0.04]	[0.06]	[0.04]	[0.07]
β_{smb}	0.01	0.03	0.14**	0.18	-0.12	0.07
	[0.03]	[0.07]	[0.07]	[0.13]	[0.07]	[0.09]
β_{hml}	-0.03	-0.01	-0.14	-0.17	-0.27***	-0.44***
	[0.04]	[0.03]	[0.1]	[0.13]	[0.05]	[0.12]
Adj. R^2	8.9%	0.2%	10.2%	10.1%	8.0%	16.4%
				corr(UMD-VW, REP)		69.50%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 7: Replicating Portfolio by Size and Time Period

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles.The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. "REP", the replicating portfolio, is the sum of the three components. "RES", the residual portfolio, is the momentum portfolio minus the replicating portfolio. "Big"/"Small" are momentum portfolios formed on stocks larger/smaller than the NYSE median. "HH" (half-half) is the portfolio that gives equal weight to the big and small momentum portfolios. "Jan. Excluded" means that all months of January are removed from the sample.

Size/Period	Big		Small		HH		HH, $\geq$1990		HH, Jan. Excluded	
	REP	RES	REP	RES	REP	RES	REP	RES	REP	RES
$\bar{r}$	0.0039***	0.0007	0.0035**	0.0050***	0.0037***	0.0029**	0.0043	0.0021	0.0030**	0.0049***
	[0.0014]	[0.0012]	[0.0014]	[0.0013]	[0.0013]	[0.0011]	[0.0031]	[0.0026]	[0.0014]	[0.0012]
$\bar{r}/\sigma$	0.0928	0.0191	0.0965	0.1192	0.0886	0.0814	0.0822	0.0566	0.0730	0.1554
info ratio	0.1320	0.0536	0.0960	0.2094	0.1213	0.1500	0.1293	0.0691	0.1173	0.1799
FF3 α	0.0052***	0.0018	0.0042***	0.0076***	0.0047***	0.0047***	0.0060**	0.0025	0.0045***	0.0055***
	[0.0012]	[0.0012]	[0.0012]	[0.0013]	[0.0012]	[0.0011]	[0.0027]	[0.0025]	[0.0013]	[0.0012]
β_{mkt}	-0.21***	0.05	-0.26***	0.02	-0.23***	0.03	-0.43***	0.10	-0.24***	0.01
	[0.06]	[0.04]	[0.06]	[0.04]	[0.06]	[0.04]	[0.12]	[0.08]	[0.06]	[0.04]
β_{smb}	0.22*	-0.15**	0.43***	-0.53***	0.32**	-0.34***	0.45	-0.32*	0.32**	-0.15*
	[0.12]	[0.07]	[0.13]	[0.09]	[0.13]	[0.08]	[0.29]	[0.16]	[0.14]	[0.08]
β_{hml}	-0.17	-0.27***	-0.06	-0.33***	-0.12	-0.30***	-0.24	-0.08	-0.17	-0.21***
	[0.13]	[0.05]	[0.12]	[0.07]	[0.12]	[0.05]	[0.27]	[0.11]	[0.12]	[0.06]
Adj. R^2	10.5%	8.8%	14.6%	26.7%	12.5%	20.1%	18.8%	6.5%	14.9%	7.6%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 8: Alternative Three-Factor Regressions

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns in the past two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. Momentum returns are then regressed on the returns of the three factor components.

	VW	Big	Small	HH
FF3 α	0.0094***	0.0071***	0.0118***	0.0075***
Alt. α	0.0039***	0.0019	0.0058***	0.0022*
	[0.0011]	[0.0012]	[0.0012]	[0.0011]
REP(MKT)	0.93***	0.91***	0.96***	0.92***
	[0.21]	[0.11]	[0.12]	[0.11]
REP(SMB)	0.44***	0.41***	0.47***	0.44***
	[0.08]	[0.08]	[0.10]	[0.08]
REP(HML)	0.82***	0.85***	0.78***	0.85***
	[0.11]	[0.10]	[0.12]	[0.1]
Adj. R^2	53.0%	49.0%	45.1%	8.0%

[ ]: Newey-West standard errors with 6 lags;

*/**/***: statistically significant at 10/5/1%

Table 9: Momentum and the Replicating Portfolio in Turbulent and Calm Periods

Turbulent periods for a factor are defined as ones in which the realized factor return lies outside of the 25th-75th percentile range of historical returns. The calm periods are the remaining periods. The "Any One" turbulent periods are ones in which the realized factor return of at least one of the factors lies outside of the 10th-90th percentile range of historical returns for that factor. UMD-HH FF3 α is the Fama-French three-factor alpha of the half-half momentum portfolio. REP α and β's are coefficient estimates from the unconditional FF3 regression of the replicating portfolio.

	Market		SMB		HML		Any One	
	Turbulent	Calm	Turbulent	Calm	Turbulent	Calm	Turbulent	Calm
UMD-HH FF3 α	0.0072***	0.0106***	0.0074***	0.0114***	0.0094***	0.0089***	0.0074***	0.0082***
	[0.0018]	[0.0013]	[0.0017]	[0.0016]	[0.0019]	[0.0013]	[0.0022]	[0.0011]
REP α	0.0066***	0.0038***	0.0044**	0.0069***	0.0052***	0.0053***	0.0057**	0.0030**
	[0.0018]	[0.0014]	[0.0018]	[0.0017]	[0.0019]	[0.0016]	[0.0023]	[0.0012]
β_{mkt}	-0.30***	-0.07	-0.20**	-0.23***	-0.31***	-0.01	-0.28***	0.01
	[0.08]	[0.05]	[0.09]	[0.07]	[0.08]	[0.06]	[0.08]	[0.05]
β_{smb}	0.01	0.39*	0.27	0.04	0.23	0.11	0.24	0.04
	[0.11]	[0.21]	[0.18]	[0.09]	[0.16]	[0.12]	[0.16]	[0.08]
β_{hml}	-0.04	-0.27**	-0.26	-0.02	-0.23	0.12	-0.27*	0.22***
	[0.18]	[0.14]	[0.18]	[0.14]	[0.16]	[0.09]	[0.16]	[0.07]
Adj. R^2	15.0%	16.4%	11.9%	10.6%	18.4%	1.4%	18.2%	15.70%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 10: Momentum and the Replicating Portfolio Based on Different Formation Periods

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns during the period specified in "Formation Period", where t is the formation month.$[t-11, t-6]$ means the most recent year omitting the most recent six months; $[t-6, t-1]$ means the most recent seven months less the most recent month. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the weighted average betas of individual stocks selected into the momentum portfolio; the weights are identical to those assigned by the momentum portfolio. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. "REP", the replicating portfolio, is the sum of the three components. "RES", the residual portfolio, is the momentum portfolio minus the replicating portfolio. "VW" and "HH" represent the value-weighted and half-half-weighted momentum portfolios, respectively.

Formation Period	$[t-11, t-6]$				$[t-6, t-1]$			
Weighting Scheme	VW		HH		VW		HH	
	REP	RES	REP	RES	REP	RES	REP	RES
$\bar{r}$	0.0047***	0.0012	0.0044***	0.0028***	0.0013	0.0004	0.0012	0.0021*
	[0.0017]	[0.0011]	[0.0016]	[0.001]	[0.0016]	[0.0013]	[0.0015]	[0.0012]
$\bar{r}/\sigma$	0.1055	0.0377	0.1030	0.0888	0.0288	0.0101	0.0260	0.0567
info ratio	0.1225	0.0704	0.1168	0.1469	0.0618	0.0454	0.0534	0.1198
FF3 α	0.0052***	0.0022**	0.0048***	0.0043***	0.0027*	0.0017	0.0023*	0.0041***
	[0.0014]	[0.0011]	[0.0014]	[0.0010]	[0.0014]	[0.0012]	[0.0013]	[0.0011]
β_{mkt}	-0.07	0.03	-0.10	0.01	-0.20***	0.00	-0.23***	-0.01
	[0.08]	[0.03]	[0.08]	[0.03]	[0.06]	[0.05]	[0.06]	[0.05]
β_{smb}	0.27***	-0.13**	0.32***	-0.28***	0.04	-0.09	0.16	-0.29***
	[0.1]	[0.06]	[0.1]	[0.05]	[0.19]	[0.13]	[0.19]	[0.1]
β_{hml}	-0.21	-0.22***	-0.15	-0.22***	-0.08	-0.28**	-0.05	-0.29***
	[0.16]	[0.05]	[0.15]	[0.05]	[0.13]	[0.11]	[0.13]	[0.11]
Adj. R^2	6.4%	6.9%	7.1%	15.1%	6.3%	7.1%	7.3%	16.6%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 11: Fama-French Alphas of the Past Return Decile Portfolios and the Replicating Portfolios

For Panels 2-3, stocks are divided into two groups based on whether their capitalizations are larger (B) or smaller (S) than the NYSE median. Within each size group, stocks are sorted into deciles by their cumulative returns in the past 12 months omitting the most recent month. Each decile portfolio is then held for one month forward. UMD columns list the FF3 alphas of the decile portfolios; REP columns list the FF3 alphas of the replicating portfolios, formed using the same methodology as the momentum replicating portfolio. Figure 5 has the graphical representation of this table.

Size	VW		Big		Small	
	UMD	REP	UMD	REP	UMD	REP
Smallest	-0.0133***	-0.0053***	-0.0081***	-0.0053***	-0.0163***	-0.0042***
	[0.0014]	[0.0011]	[0.0012]	[0.0011]	[0.0014]	[0.001]
2	-0.0071***	-0.0029***	-0.0046***	-0.0027***	-0.0087***	-0.002***
	[0.0011]	[0.0008]	[0.0009]	[0.0008]	[0.001]	[0.0007]
3	-0.0052***	-0.0028***	-0.0017**	-0.0029***	-0.0049***	-0.001*
	[0.0009]	[0.0007]	[0.0007]	[0.0007]	[0.0008]	[0.0006]
4	-0.0024***	-0.0010**	-0.0020***	-0.0009*	-0.0039***	-0.0012**
	[0.0007]	[0.0005]	[0.0006]	[0.0005]	[0.0007]	[0.0005]
5	-0.0017***	-0.0008*	-0.0015**	-0.0008*	-0.0039***	-0.0011**
	[0.0006]	[0.0004]	[0.0006]	[0.0004]	[0.0007]	[0.0005]
6	-0.0012**	-0.0002	0.0005	-0.0003	-0.0017***	-0.0001
	[0.0005]	[0.0003]	[0.0006]	[0.0003]	[0.0006]	[0.0005]
7	0.0009	0.0009**	0.0010*	0.0009**	-0.0009	0.0003
	[0.0006]	[0.0004]	[0.0005]	[0.0004]	[0.0006]	[0.0005]
8	0.0023***	0.0014***	0.0019***	0.0012***	0.001	0.0009
	[0.0006]	[0.0005]	[0.0007]	[0.0005]	[0.0006]	[0.0006]
9	0.003***	0.0025***	0.0025***	0.0023***	0.0023***	0.0017***
	[0.0007]	[0.0007]	[0.0008]	[0.0007]	[0.0007]	[0.0006]
Biggest	0.0051***	0.0044***	0.0054***	0.0041***	0.0042***	0.0030***
	[0.0012]	[0.0011]	[0.0012]	[0.0011]	[0.001]	[0.001]

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

Table 12: Momentum Replicating Portfolios Based on Various Beta Estimation Windows

At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns during different windows. The first three scenarios all involve two-year windows; they are nonoverlapping periods starting at 47 months prior to, 23 months prior to and one month ahead the formation month, respectively. The last scenario involves a longer window of the most recent five years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the value-weighted average beta of stocks in the long portion minus that in the short portion. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. The replicating portfolio is the sum of the three components. In the first, second and fourth scenario, the replicating portfolio is a tradable strategy.

REP	[-4, 2]	[-2, 0]	[0, 2]	[-5, 0]
$\bar{r}$	0.0021***	0.0041***	0.0019	0.0025**
	[0.0008]	[0.0014]	[0.0013]	[0.0012]
$\bar{r}/\sigma$	0.0805	0.0964	0.0448	0.0732
info ratio	0.0857	0.1382	0.1426	0.1117
FF3 α	0.0022***	0.0056***	0.0052***	0.0037***
	[0.0008]	[0.0012]	[0.0011]	[0.001]
β_{mkt}	-0.09**	-0.21***	-0.23***	-0.17***
	[0.04]	[0.06]	[0.07]	[0.06]
β_{smb}	0.08	0.18	-0.08	0.16*
	[0.07]	[0.13]	[0.08]	[0.1]
β_{hml}	0.05	-0.17	-0.41***	-0.13
	[0.06]	[0.13]	[0.13]	[0.12]
Adj. R^2	3.3%	10.1%	27.8%	9.4%
ρ (., umd)	52.4%	71.1%	84.4%	71.8%

[ ]: Newey-West standard errors with 6 lags;

*/**/***: statistically significant at 10/5/1%

Table 13: Fama-French Factor Autoregressive Portfolios

The correlations are between a factor's average return this month and return in the past year (omitting the previous month). Outliers are defined as past returns whose absolute values are above 4% for market and 3% for SMB and HML. The autocorrelation portfolio is a factor portfolio with weights equal to the past returns; at time t it has a return of $r_{t-12,t-2}r_t$. All point estimates and standard errors are 100 times their actual values.

	Full Sample			Outliers Winsorized		
	MKT	SMB	HML	MKT	SMB	HML
$\rho\left[r_{t-12,t-2}, r_t\right]$	1.5%	6.5%**	0.9%	6.9%**	10.5%***	7.6%**
	{0.64}	{0.04}	{0.78}	{0.03}	{0.00}	{0.02}
$\rho\left[r_{t-12,t-2}, r_t^2\right]$	-24.1%***	4.1%	-6.2%**	-17.8%***	4.9%	-12.8%***
	{0.00}	{0.04}	{0.05}	{0.00}	{0.13}	{0.00}
FF3 α ×100	0.0125**	0.0018	0.0014	0.0077*	0.0023**	0.0027**
	[0.0053]	[0.0013]	[0.0019]	[0.0041]	[0.0011]	[0.0013]
$\beta_{mkt}\times 100$	-0.62	-0.04	-0.18	0.01	-0.02	-0.21*
	[0.56]	[0.04]	[0.13]	[0.32]	[0.03]	[0.11]
$\beta_{smb}\times 100$	0.5	0.48***	0.46**	0.22	0.39***	0.29**
	[0.38]	[0.16]	[0.19]	[0.21]	[0.14]	[0.14]
$\beta_{hml}\times 100$	-1.28	0.04	0.09	-0.13	0.01	-0.25
	[0.99]	[0.09]	[0.34]	[0.22]	[0.07]	[0.3]
Adj. R^2	10.28%	12.93%	4.78%	8.94%	12.92%	9.16%

[ ]: Newey-West standard errors with 6 lags; */**/***: statistically significant at 10/5/1%

{ }: p-values for $H_0:\rho=0; H_a:\rho\neq 0$

Table 14: Reconstruction of Momentum's FF3 Alpha from Factor Structure Autocorrelations

The alpha of the replicating portfolio is the sum of the alphas of the three factor components. Each component is roughly a $M\times$ magnified version of the factor autocorrelation portfolio, which is the factor itself scaled by its past returns, i.e., $F_{t-12,t-2}F_t$. The alpha of the factor autocorrelation is in turn approximately the sum of two components, the autocovariance of the factor $\hat{E}\left[F_{t-12,t-2}F_t\right]$and the covariance between the factor's squared returns and past returns $\hat{E}\left[F_{t-12,t-2}F_t^2\right]\times\left\{E\left[F_t\right]/\sigma^2\left[F_t\right]\right\}$. The factor covariances are estimated in a robust regression limiting the influence of outliers. The magnifier is estimated in a robust regression of momentum beta on a factor on the factor's past returns, also limiting the influence of outliers. The alphas of the autoregressive portfolios are similar to those in the right panel of Table 13.

	$\rho\left(r_{t-12,t-1}, r_t\right)$	$\rho\left(r_{t-12,t-1}, r_t^2\right)$	$\text{cov}\left(r_{t-12,t-1}, r_t\right)$	$\text{cov}\left(r_{t-12,t-1}, r_t^2\right)$
MKT	6.9%	-17.8%	4.93E-05	-1.50E-05
SMB	10.5%	4.9%	2.88E-05	1.16E-06
HML	7.6%	-12.8%	2.12E-05	-2.95E-06

	Autocorr.	Leverage	Autoreg. α	Magnifier (M)	α
MKT	5.08E-05	2.86E-05	7.94E-05	17.97	0.0014
SMB	2.97E-05	-2.61E-06	2.71E-05	61.06	0.0017
HML	2.18E-05	9.06E-06	3.09E-05	55.32	0.0017
				Momentum α	0.0048

Table 15: Momentum Replicating Portfolio relative to the FF3 Factors + Financial Distress Factor

The financial distress factor (FDF) is a long-short portfolio formed by sorting stocks on their predicted delisting probabilities, which are estimated from a logistic regression model using firm characteristics. At the end of each month, individual stock betas are estimated by regressing monthly excess returns on the Fama-French three-factor returns and the financial distress factor during the most recent two years. The cross-section of betas is winsorized each month at the 1st and 99th percentiles. The momentum portfolio betas are the value-weighted average beta of stocks in the long portion minus that in the short portion. Each factor component of the replicating portfolio is the factor itself scaled by momentum's beta on that factor. The replicating portfolio is the sum of the four components.

	UMD-HH		REP (FF3)		REP(FF3+FDF)	
$\bar{r}$	0.0066***		0.0049**		0.0064**	
	[0.0022]		[0.0021]		[0.0029]	
$\bar{r}/\sigma$	0.1516		0.1092		0.1266	
info ratio	0.2045	0.1461	0.1269	0.1464	0.2005	0.1885
FF3 α	0.0086***	0.0063**	0.0054***	0.0063**	0.0093***	0.0087***
	[0.0018]	[0.0029]	[0.0021]	[0.0025]	[0.0026]	[0.0029]
β_{mkt}	-0.18**	-0.11	-0.19*	-0.28**	-0.38***	-0.37***
	[0.09]	[0.08]	[0.10]	[0.11]	[0.10]	[0.11]
β_{smb}	0.11	0.27*	0.34	0.23	0.30	0.33
	[0.13]	[0.15]	[0.23]	[0.23]	[0.28]	[0.25]
β_{hml}	-0.35**	-0.30	-0.14	-0.25	-0.32	-0.30
	[0.17]	[0.18]	[0.22]	[0.25]	[0.23]	[0.25]
β_{fdf}		0.44**		-0.04		0.08
		[0.20]		[0.16]		[0.20]
Adj. R^2	7.0%	8.7%	7.3%	8.7%	14.0%	13.9%
ρ (., UMD-HH)			68.3%		68.8%	

[ ]: Newey-West standard errors with 6 lags;

*/**/***: statistically significant at 10/5/1%