Certification vs. Credibility in the Presence of Soft Information*

Jeremy Bertomeu and Iván Marinovic

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Abstract

This paper studies voluntary certification when (a) there is uncertainty about managerial propensity to report truthfully, (b) some components of the firm's value may be certified for a cost ("hard"), (c) other components may be disclosed but not certified free of misstatements ("soft"). We establish that untruthful managers are more likely to certify hard information and that, among truthful managers, those with more favorable soft information also certify more. Even if certification is costless, unraveling to a complete certification of the hard information may not occur. We develop several testable predictions linking certification to trust, reporting quality, the likelihood and magnitude of frauds, and market prices. The model has many natural applications, including credit ratings, press releases, auditing, going dark and voluntary appraisals.

Keywords: cheap talk, disclosure, certification, trust, markets.

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1 Introduction

Most market instances of strategic communication involve the following ingredients: the information is multidimensional; some pieces of information can be certified at a

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cost; some pieces of information cannot be certified and therefore they can only be communicated if the seller has some credibility; there is uncertainty about the credibility of the seller, i.e., about the likelihood that a seller who faces conflicting economic incentives reports truthfully.

The interaction between these two types of information presents several obvious questions of interest. Should the decision to certify hard information depend on any soft information, *even if* that soft information cannot be certified? Should an untruthful sender, i.e., one with more reporting discretion, certify more than a truthful sender to curtail this discretion, *even when* her untruthfulness is not common knowledge? And, if the answer to either of these questions is positive, what does certification imply about the sender's credibility, her reporting choices, or how the receiver should evaluate uncertified soft reports.

We formally study strategic communication in this context. Since most market transactions can be described in these terms our analysis has numerous applications. Below we list several applications, beginning with what we use as our baseline interpretation.

Publicly traded firms periodically release information to investors in the capital market. Sometimes the information presents detailed evidence about tangible hard assets whose value can be certified by an outside party and, at other times, the information reflects management estimates (e.g., brands, goodwill, patents, etc.) whose true value is hard to objectively verify. Further, some of the information is disclosed through formal channels, such as financial statements and other regulatory filings, which are subject to independent audits and strict penalties for perjury. Sometimes, the information is distributed through informal channels such as press releases, conference calls, conversations with analysts, investor meetings or even advertisements in news outlets (e.g., Frankel, Johnson and Skinner 1999, Bushee, Core, Guay and Hamm 2010).

Similarly, when a company issues debt, it may choose to certify the issuance by hiring a certified rating agency (Beaver, Shakespeare and Soliman 2006, Gul and Goodwin 2010) The rating is, however, a very partial assessment of the issuance: the rating certifies the issuer's probability of default but it does not assess the issuer's profitability if default does or does not occur. The issuer has two options: either hire the rating agency and supplement the rating with additional financial information, or not hire the rating agency and provide self-reported information (in which case the issue is usually privately placed at priced at a higher risk.) Furthermore, a positive rating on a debt issue will certify some of the firm's assets and convey positive information to other equity holders.

Another important choice is whether a firm should go public or remain private, or even delist its stock (Bushee and Leuz 2005, Engel, Hayes and Wang 2007). While such a choice has many implications, an important one is that public firms should follow the disclosure requirements of regulators and the exchanges where the stock is traded such as, for example, issuing quarterly financial statements with a number of designated items reviewed by an external auditor. A public firm that delists its stock from a stock exchange may, under certain circumstances, suspend its reporting obligations.

To fix ideas, assume the seller is a firm's manager who has private information about the firm's tangible assets (hard) and the firm's customer satisfaction (soft) –where the latter is presumably correlated with future revenue. The manager wishes to sell her firm for the highest possible price. But, given the obvious conflict of interest, her credibility is imperfect: the market knows that the manager may have discretion to manipulate any uncertified information to increase the market price. Given the risk of misreporting, the market discounts information that seems "too good to be true." To overcome this skepticism, the manager can hire a certifier. The certifier will verify the existence of certain hard assets and the valuation methods that have been used. But the certifier cannot certify soft assets such as customer satisfaction (or, for that matter, many of the firm's intangible assets.) Any uncertified assets, whether hard or soft, are informally communicated and are thus subject to manipulation by untruthful managers.

In this context, we show that untruthful managers certify *more*, relative to managers that must report truthfully. While this may seem counter-intuitive at first blush (since it is the manager with the greatest discretion who is willing to reduce that discretion the most), the intuition is tied to the substitution between formal certification and informal communication. A soft report claiming low total assets is a credible signal that the manager is truthful and thus does not require any extra certification. By contrast, a soft report claiming high total assets is discounted for the probability that this report could have been issued by an untruthful manager. Since certification removes part of this discount, it is of greater value to firms that would, absent certification, report higher total assets. It then follows that a truthful manager observing high asset values certifies more than truthful managers observing low asset values. The same intuition applies to untruthful managers: while they do not necessarily observe high total assets, they report as if; hence, untruthful managers are the most willing to certify hard information.

This yields several novel empirical implications which, to our knowledge, have not yet been tested. First, informal communication creates an informational channel whereby the decision to certify conveys information about assets that cannot be certified. The decision to certify hard assets should be positively correlated with the value of soft assets because, in equilibrium, the decision to certify is a function of the entire asset

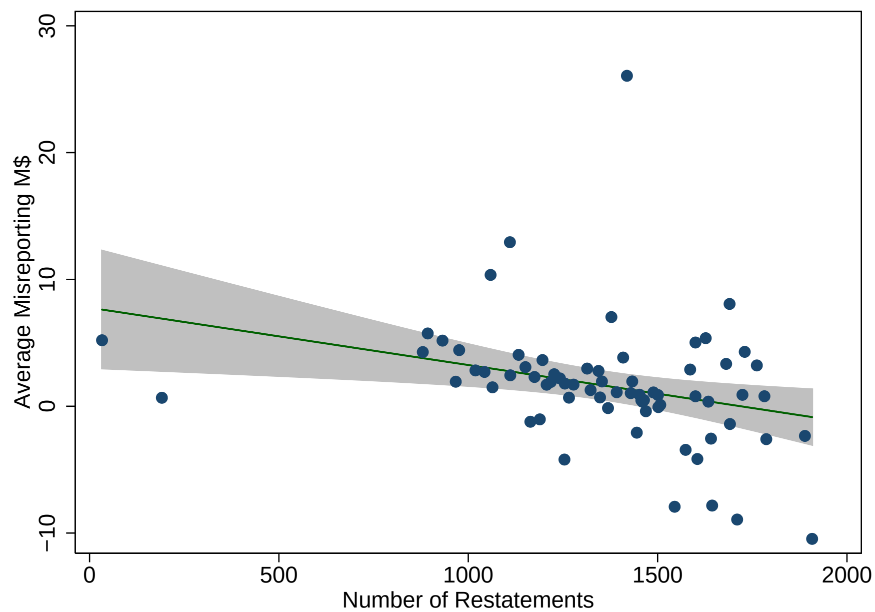

Figure 1: *The graph is based on the entire sample of restatements recorded in the database UNRESTATED COMPUSTAT for the period prior to the year 2002. The average misreporting is calculated as the quarterly average across restating firms of the difference between net income as originally reported (ibqr) and its restated value (ibq).*

value (including the non-certifiable components.) Second, since untruthful managers are more likely to certify, the credibility of a manager in the certified market should be lower than in the uncertified market and frauds will be more likely to occur in the certified market. Third, frauds (when they occur) are greater when investors perceive the manager as being more likely to be truthful, i.e., when there is greater overall trust in the market. Hence, frauds should be greatest, though less frequent, in the uncertified market and, more generally, the average size of frauds should be negatively related to their likelihood. Figure 1 provides further anecdotal evidence in the context of financial restatements and is consistent with average restatements being greater when restatements are less frequent.

1.1 Detailed overview of the results

A detailed overview of the results follows. We provide an economic analysis of competing modes of communication, where formal and informal communication are alternative means through which firms may convey information to the market. This decision is determined by three underlying factors: managerial credibility, the cost of certification and the volatility (or uncertainty) relating to soft and hard information.

We establish that trust is a central factor that affects both certification and reporting frauds. Unsurprisingly, high confidence environments feature a lesser reliance on formal certification. We also find that higher confidence increases the average magnitudes of frauds when they occur, because untruthful managers exploit confidence by reporting more aggressively. Since our model assumes rational well-calibrated expectations, this is in sharp contrast to the conventional argument that large frauds occur because unsophisticated investors are overly optimistic and, on average, fooled by management. This has further implications for market regulations which, in practice, tend to follow public pressures after the discovery of large-scale frauds (e.g., Enron, Worldcom.) A few large-scale frauds are the natural consequence of high levels of credibility where additional regulatory oversight is, perhaps, least likely to be necessary.

An increase in certification costs reduces firms' propensity to certify. Interestingly, the reduction is more pronounced for untruthful managers (since from an ex-ante standpoint, untruthful managers tend to certify more frequently) thus increasing the odds of an untruthful manager in the uncertified market. Yet, because some higher-value truthful firms also shift to the uncertified market, the increase in certification costs encourages more aggressive reporting by untruthful firms in that market. Put together, we find that greater certification costs raises both the likelihood and magnitude of frauds in the uncertified market.

The effect of uncertainty on certification and frauds depends if the volatility relates to hard or soft information. Higher volatility of the hard information increases demand for certification, in particular for the untruthful managers who, as we noted earlier, are the most willing to certify. This causes an increase (decrease) of the likelihood of misreporting in the certified (uncertified) market. In comparison, higher volatility of the soft information affects credibility in the certified and uncertified market symmetrically and does not alter the probability of certification or frauds; however, such greater volatility increases maximal prices in both markets and the probability of frauds.

As an aside, the analysis provide a novel argument as to why the well-known unravelling principle (Grossman and Hart (1980)) may not hold. Even if the cost of certification is zero, there can be equilibria with no certification when certification indicates unfavorable information about any remaining soft information. In other cases, a weaker version of unravelling may apply: whenever managerial credibility is sufficiently high, a complete revelation of the information can occur through informal communication channels and without any formal certification.

We examine an extension of the model to imperfect competition in the certification market. Certification costs are determined endogenously as the fee posted by a monopolistic certifier. Contrary to standard disclosure models (see, e.g., Verrecchia, 1990, Levin, 2001), we show that greater volatility does not result in more certification. The certifier accommodates changes in volatility by raising his fees exactly in the amount required to keep the probability of certification constant. The profit of the certifier increases in situations of low market confidence and high volatility.

The social externalities of disclosure are then illustrated in a version of the model in which the manager makes an initial investment. The value of investment is partially offset by the misreporting of untruthful managers (who always invest.) Absent certification costs, greater volatility would always raises the value of investment. Yet, volatility also increases the demand for certification, leading to greater certification costs. At the fee that maximizes the profit of the certifier, the expected value of the firm decreases when volatility increases. This would suggest that uncertainty is undesirable (desirable) to the firm when the certification market is monopolistic (competitive).

1.2 Literature Review

The main contribution of the paper is to provide a theory of strategic communication where not only the message but also the *level of verifiability* of the communication is at the sender's discretion. To understand this, a contrast with the disclosure literature might help. The disclosure literature studies the circumstances under which the sender publicly unveils verifiable information. In the standard models of disclosure (e.g., Verrecchia 1983, Dye 1986, Jung and Kwon 1988, Verrecchia 1990, Jorgensen and Kirschenheiter 2003, Hughes and Pae 2004, Einhorn and Ziv 2008), the disclosure is fully credible and about the entire cash flow of the firm. We extend this literature along two dimensions, in that our environment feature partially, but not entirely, verifiable cash flows and some supplementary unverifiable information may be issued.

Absent certification, communication is possible in our model by the receiver's perception that the sender is truthful with positive probability. In the absence of certification, the sender engages in informal communication or cheap talk (Crawford and Sobel 1982, Fischer and Stocken 2003, Ottaviani and Sorensen, 2006). The cheap talk literature assumes that the sender's objectives are partly, but not completely, misaligned with those of the receiver. Like Morgan and Stocken (2003), we assume that the sender is privately informed about the extent of this misalignment. In particular, we assume that under certain circumstances (that only the sender observes) the sender is bound to tell the truth. The uncertainty of the receiver about the credibility of the sender together with the possibility to certify part of the information, enables the sender to partially overcome the receiver's skepticism, enabling informative communication even when incentives are likely to be extremely misaligned. By considering an alternative process to make the information hard and verifiable, the model allows us to measure the costs of lack of credibility and alternative means of disclosure that firms use to overcome these costs.

2 Model

This is a model of strategic communication between the seller of a good or service and its prospective buyers. For the purpose of this study, the good may be labor services, a product, or securities sold in a financial market but, in order to facilitate the exposition, we use the interpretation of a firm whose stock is sold in a competitive financial market. We refer to the seller as the manager, to the buyers as the market and to the item as the firm (as in Grossman and Hart 1980, the sale may be an initial public offering.) The firm is priced in a competitive market and the manager maximizes the sale price of the firm.

Let $\pi \in [\underline{\pi}, \overline{\pi}]$ be the value of the firm, and assume that this value depends on two independent pieces of information that are privately observed by the manager: a "hard" piece h that the manager can certify and a "soft" piece s that the manager can only communicate to the market informally.

A1. *The value of the firm is additively-separable in the hard and soft information, i.e.* $\pi = h + s$.

The purpose of the additive structure is to focus a research design in which soft information does not increase or decrease the value of assets that could be certified. We assume no direct linkages between certification and soft information except those driven by strategic issues arising from informational asymmetries.

A2. *The cost of certifying* h *is* $c > 0$. *When* h *is certified we assume that* h *becomes public information.*

One can think of certification as any process through which an independent party attests the value of h, such as hiring an auditor to verify financial statements or using the services of a credit rating agency.[1] The manager always has the option not to certify h, in which case both h and s, and thus π, are disclosed informally, i.e., as information that can be potentially misreported by the manager. What we call an *uncertified report* could

[1]As Lang and Lundholm (1993) point out, the notion that preparation costs have a fixed component underlies much of FASB's and SEC's consideration of firm size in disclosure requirements. They also note that the SEC has separate 10K and 10Q filing requirements for small firms, labeled 10KSB and 1OQSB, to lighten the burden of accessing the equity markets.

be thought of as the manager issuing a press release, or providing an aggregate number with little supporting (proprietary) evidence that the disclosure is indeed appropriate.

A3. *h and s are independently distributed and satisfy: (i)* $E(h) = E(s) = 0$*, (ii) h admits a log-concave distribution* $F_h(\cdot)$ *with positive density* $f_h(\cdot)$ *over* $[-\sigma, \sigma]$*, (iii) s has a binary distribution with support over* $\{-q, q\}$*.*

The assumption that the distribution of s is binary is only made to simplify the exposition. In Section 7 we consider the case in which the distribution of s is continuous. Following Benabou and Laroque (1991) and more recently Chen (2011), we assume that the manager's reporting discretion is represented by a binary random variable $\tau \in \{0, 1\}$. $\tau = 1$ indicates that the manager must report truthfully any private information he is aware of, and $\tau = 0$ indicates that the manager has reporting discretion over any information that is not certified.[2] In other words, under $\tau = 0$, the manager's communication, absent certification, is cheap talk.[3]

A4. *Reporting discretion,* τ*, is independent from h and s and is such that* $\Pr(\tau = 1) = \gamma$*. The realization of* τ *is known to the manager but not to the market.*

We refer to $\theta \equiv \frac{\gamma}{1-\gamma}$ as the firm's credibility, i.e., the likelihood that the manager must be truthful. θ could capture things such as the manager's honesty or ethical standards, the quality of the firm's control system, the effectiveness of market institutions, the existence of incentives to maximize interim stock prices, etc. For example, if we interpret truthfulness as an ethical standard, our working assumption is that some managers are more ethical than others and are not willing to openly misrepresent information.[4] This is assumption is motivated by a number of experiments in this area which indicate that certain individuals are more averse (or able to) to lie than others (Hurkens and Kartik 2009, Ederer and Fehr 2009). A different interpretation is that some managers are the

[2] Chen (2011) note that there is a long tradition, going back to at least Kreps and Wilson (1982) and Milgrom and Roberts (1982), of introducing behavioral types to explain phenomena that cannot be explained by fully strategic players. That the players may be nonstrategic is consistent with experimental and other empirical evidence. For instance, experimental studies by Forsythe et al. (1999) show that in cheap-talk games, some sender subjects displayed a tendency to reveal the true state even when they have a clear incentive to lie.

[3] Benabou and Laroque (1992) argue that $\tau = 1$ can be interpreted either behaviorally –an honest type is one who strictly adheres to a code of ethics under which he pledges to always tell the truth –or in terms of payoff uncertainty. A manager faces penalties if caught lying, but only he knows his probability of escaping discovery. The public is thus uncertain whether the expected penalty is sufficient to deter lying.

[4] Note that we take here very seriously the idea that (some) agents' actions may be restricted by ethical standards, i.e, there may be reasons to trust an economic agent for financial markets to function properly (Zingales 2009). A related explanation is developed in Fischer and Huddart (2008) who model such behaviors as emerging endogenously from a group norm within the organization. In this context, it would be difficult for external parties (who are not employees) to directly peek into these ethical standards.

object of tighter audits (whose extent cannot be credibly communicated externally) or, within the conduct of their business, have engaged in transactions that are easier to audit or have been the object of internal controls.[5]

The market is competitive, and the firm is priced at the expected value conditional on all publicly available information.

A5. *There is significant uncertainty about the firm's soft information, that is q is sufficiently large ($q \geq \overline{q}$).*[6]

We focus here on the case in which soft information is important relative to hard information. This assumption provides more tractability to the model and seems reasonable for our baseline application given that, in the vast majority of cases, the value of a firm depends more on forward-looking measures (e.g., brand images, research projects, customer satisfaction, etc.).[7] From a conceptual standpoint, the assumption is useful to remove less interesting cases in which soft information is a second-order effect and thus where the main forces become similar to a single-dimensional costly certification. We will also point out several results that depend on A5 and, for completeness, discuss $q < \overline{q}$ in Appendix B.

The timeline of the model contains the following events. First, the manager privately observes reporting discretion, τ, and the realization of hard and soft information h and s. Second, she decides whether to (i) certify h for a cost c and make an informal report about s, or (ii) not certify any information and report the entire value π informally. Third, upon observing the report and certification choice, buyers compete to buy the firm and the price of the firm's assets is set equal to the expected value conditional on all public information.

[5]One may argue that it is not in the best interest of the first generation of shareholders to require honest reporting (since a honest manager does not lie to increase the market price.) However, ethical standards are unlikely to be observable by the board and shareholders who are not selling do not benefit from misreporting. Within the interpretation of audit quality, γ may be interpreted as the ex-ante quality of internal controls which may required by law. With some probability, this control system may be effective (in which case the manager may not misreport) or ineffective (in which case the manager may misreport information.)

[6]The required bound $\overline{q}$ is formally given in Appendix A, Eq. (28); however, for our current purpose, the assumption is much better understood by viewing the uncertainty relating to soft information as sufficiently important relative to the uncertainty relating to hard information.

[7]Perhaps Market-to-Book ratios provide a rough approximation of the relative importance of firm's soft information. According to McNichols, Rajan and Reichelstein (2010) the average Market-to-Book ratio across all firm-year observations with available Compustat data is 2.5.

2.1 Beliefs, Strategies and Equilibrium Definitions

Any equilibrium consists of a certification strategy, a reporting strategy for the untruthful manager, and a pricing function. These objects are formally defined below.

Reporting Strategy Let $\omega = (h, s, \tau) \in \Omega$ denote the manager's private information. In general, one can think of her message as a two-dimensional report about (h, s) – plus a binary certification decision $d : \Omega \to \{0, 1\}$, where $d = 1$ means that h is certified− satisfying that:

(a) conditional on $\tau = 1$, any report must be truthful (in a sense specified below), and

(b) conditional on $\tau = 0$:

  i. if the manager chooses to not certify h, then she can report anything −her report must only belong to $[\underline{\pi}, \overline{\pi}]$;

  ii. if the manager chooses to certify h then h becomes publicly observable, but she can still lie about the soft component s.

It is convenient (and without loss of generality) to simplify the description of the report by assuming that when the manager does not certify h she reports the total value π and when she does certify h she only reports the value of s (as the value of h becomes public under certification.)

Some notation is in order. Let x_d denote the manager's report conditional on her certification choice, where $d \in \{0, 1\}$. When $\tau = 1$ the manager must be truthful, hence by assumption $x_0 = \pi$ and $x_1 = s$. On the other hand, when $\tau = 0$, $x_0 = r_0$ and $x_1 = r_1$, where r_0 denotes the manager's report under no certification and r_1 denotes her report under certification. Notice that when $\tau = 0$ the manager may choose to randomize her reports, so both r_0 and r_1 may be random variables. We represent this randomization by two functions $\varphi_0(\cdot)$ and $\varphi_1(\cdot)$, where the former is the p.d.f. of r_0 and the latter is the probability mass function of r_1. In the following, we refer to φ_0 and φ_1 as the manager's reporting strategy.

Price System The market's information set I may be either $I = \{d = 0, x_0\}$ or $I = \{d = 1, h, x_1\}$. The price $P(I)$ is then defined by the following conditional expectation

$$P(I) = E(\pi | I) - cd \tag{1}$$

Since the untruthful manager chooses the report to maximize $P(I)$, the relevant price for that manager is the maximal price that can be achieved conditional on the certification choice d. Accordingly, we denote $p_0 = \sup_{x_0} P(\{d = 0, x_0\})$ and $p(h) = \sup_{x_1} P(\{d = 1, h, x_1\})$.

Equilibrium Definition We use the equilibrium concept of Perfect Bayesian ("PBE"), as defined below.

Definition 1 *A PBE consists of certification strategies, $d(\cdot) : \Omega \to \{0, 1\}$; reporting strategies for the untruthful manager, $\varphi_d(\cdot) \in \Delta$; and a pricing function, $P(\cdot)$, such that:*

(a) Given $\tau = 0$, (h, s) and $P(\cdot)$, the manager's reporting and certification choices, $\{d, \varphi_d\}$, maximize $P(I)$.

(b) Given $\tau = 1$, (h, s) and $P(\cdot)$, d maximizes $P(I)$ subject to $x_d + d * h = \pi$.

(c) On the equilibrium path the pricing function is computed according to Bayes' rule as

$$P(I) = E(\pi|I) - c * d.$$

Most of the elements of the equilibrium are standard. Condition (a) states that the untruthful manager certifies and reports optimally. Condition (b) states that the truthful manager certifies optimally and reports truthfully. Condition (c) states that prices are computed according to Bayes' rule conditional on conjectured certification and reporting strategies. The next two sections complete this definition by specifying off-equilibrium beliefs.

3 Equilibrium without certification

We develop a benchmark type of equilibrium in which no firm certifies. This will serve two main purposes: first, to illustrate that soft information can affect some common results in the disclosure literature and, second, to lay out the simplified outline of a more general argument to be used in later sections.

Consider how prices are determined when there is no certification. The untruthful manager will always make reports that maximize the market price, and thus one may define $\hat{p}$ as the market price that is attained after any such report. The market infers that any report x_0 strictly below $\hat{p}$ is made by the truthful manager and thus views it as entirely credible, i.e., $P(\{d = 0, x_0\}) = x_0$. In such equilibrium the pricing function is given by

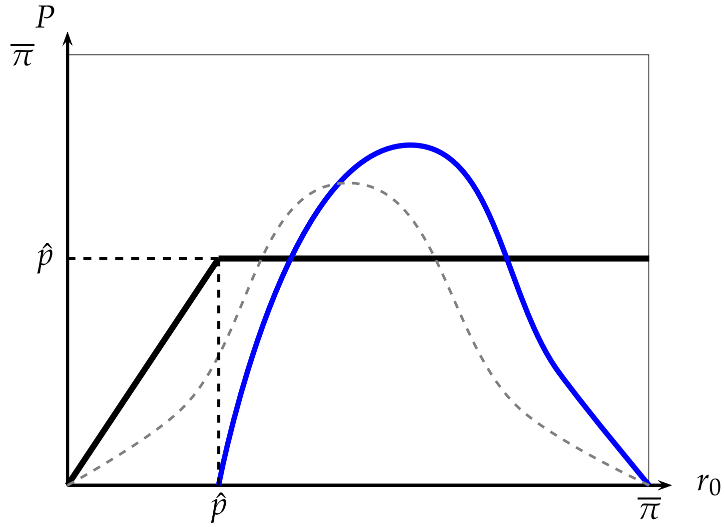

Figure 2: *Equilibrium. The blue p.d.f represents the reporting strategy of the untruthful manager* φ_0 *whereas the dashed p.d.f. represents the distribution of the firm's true value* π*.*

$$P(\{d=0,x_0\}) = \min(x_0,\hat{p}). \tag{2}$$

The upper bound of the price function $\hat{p}$ must be consistent with Bayesian updating. Specifically, any report $x_0 \geq \hat{p}$ may have been issued by a truthful manager with $\pi \geq \hat{p}$, or an untruthful manager who always reports $x_0 \geq \hat{p}$ regardless of the information received. In the former case the manager will generate, on average, $E\left(\pi|\pi \geq \hat{p}\right)$. In the latter case, the untruthful manager will generate on average $E\left(\pi|\tau=0,x_0\right) = E\left(\pi\right)$. If the untruthful issues with positive density all the reports above $\hat{p}$, then one can obtain the maximum price, $\hat{p}$, by Bayes' rule from:

$$\hat{p} = \theta E\left(\pi - \hat{p}|\pi > \hat{p}\right) \Pr\left(\pi > \hat{p}\right). \tag{3}$$

Lemma 2 *Equation* (3) *has a unique solution* $\hat{p} \in \left(E\left(\pi\right), \overline{\pi}\right)$*. Furthermore,* $\lim_{\theta\to\infty} \hat{p} = \overline{\pi}$*.*

Lemma 2 indicates that the report is not viewed as entirely credible if it is above the threshold $\hat{p}$. All disclosures below $\hat{p}$ are always credible and, for such events, certification would serve no purpose.

To verify that no certification is indeed an equilibrium, we consider the beliefs required to induce managers not to certify h under all possible circumstances. Of course, the most unfavorable of such beliefs is when a certification is perceived to have been made by an untruthful firm with $s=-q$. Since one may question such beliefs as being too extreme, we shall here illustrate the argument with a less adverse belief in which the market perceives a certifying firm to have value equal to $h + E\left(s\right) - c$. That is, the market does not learn anything about the firm's soft information from the deviation.

Proposition 3 *There is an equilibrium with no certification if and only if θ is sufficiently large. In this equilibrium, the reporting strategy of the untruthful manager is given by:*

$$\varphi_0(r_0) = \theta \frac{r_0 - \hat{p}}{\hat{p}} f_\pi(r_0) \text{ for } r_0 \in [\hat{p}, \overline{\pi}] \tag{4}$$

where $f_\pi(\cdot)$ denotes the p.d.f. of π. The maximum price $\hat{p}$ increases in θ, q and σ.

To sustain the equilibrium without certification, the manager must be sufficiently credible. Interestingly, since $\hat{p}$ converges to $\overline{\pi}$ as credibility becomes very high, the no-certification equilibrium may exist even if $c = 0$, in contrast to the unravelling theorem (see, e.g., Grossman, 1981, Milgrom, 1981). In this model, the reason why zero certification can be sustained even when it is costless is that certification alone does not confirm the entire value of the firm and may lead to unfavorable beliefs about the remaining soft assets of the firm. It is also worth noting that if we allow for more unfavorable beliefs about deviant firms, the equilibrium can be sustained for smaller values of θ.

For the market, observing a report x_0 when x_0 is greater than $\hat{p}$, is equivalent to observing the seemingly coarser report that $x_0 \geq p_0$. One would then think that the manager could agree to use coarser language (i.e., to report an interval rather than a point estimate) but note that this is not a possibility. Unlike standard cheap talk models where the language is indetermined, in our setting the only possible language is the natural one.[8] To escape detection, the untruthful manager must use a randomized reporting strategy φ_0 as specified by Eq. (4). For any $r_0 \geq \hat{p}$, the true reporting density f_π is altered by two terms. The first term θ represents the ex-ante credibility of firm: as managers are perceived as more truthful, the untruthful manager reports more aggressively, in the sense that $E(r_0)$ increases. The second term $(r_0 - \hat{p})/\hat{p}$ represents an additional distortion over high reports. That is, *even though these reports yield the same price in equilibrium*, the untruthful manager reports relatively higher reports than the truthful manager.

Before proceeding to the next section, it is interesting to consider the limit as the probability of truthfulness goes to zero when certification is not available ($c \to \infty$), because this case corresponds to the standard cheap talk literature, in which all senders

[8] Since, thuthful managers by assumption use the natural language (the support of f_π), the untruthful must always use the same language, because otherwise she would reveal her identity. In other words, the support of φ_0 has to be a subset of the support of f_π.

are strategic and there is no payoff uncertainty. In this model, the maximum price $\hat{p}$ converges to the prior expected $E[\pi]$ as credibility shrinks, $\theta \to 0$. As in cheap talk models, the extreme conflict of interest means that no information transmission can take place. Unlike cheap talk models, the reporting strategy of the sender is not indetermined as $\theta \to 0$. In fact, the reporting strategy of the untruthful manager converges to a unique non-degenerate distribution

$$\lim_{\theta \to 0} \varphi_0 (r_0) = \psi \times r_0 \times f_\pi (r_0) \text{ for } r_0 \in [0, \pi]$$

where

$$\psi = E (\pi | \pi > 0) \Pr (\pi > 0)$$

This result suggests that the language indetermination that is typical in cheap talk models can be removed by introducing a small measure of truthful senders. This assumption can also be a powerful way of reducing the multiplicity of equilibria arising in cheap talk models.

4 Equilibrium with certification

The equilibrium without certification is predicated on investors believing that untruthful managers are more likely to send off-equilibrium messages. Yet, these beliefs are somewhat arbitrary and do not satisfy standard refinements such as Grossman and Perry (1990) PSE concept or Farrell's (1993) neologism proofness.[9] So in the sequel we characterize an equilibrium with positive probability of certification which under reasonable conditions (see Appendix A.) is the unique equilibrium of the game. [10]

Proposition 4 *There is an equilibrium with certification characterized as follows.*

(i) The maximal non-certified price is given by:

$$p_0 = k - c + z \tag{5}$$

[9] Grossman and Perry (1990) define perfect sequential equilibrium PSE as follows "A perfect sequential equilibrium is supported by beliefs p which prevent a player from deviating to an unreached node, when there is no belief q which, when assigned to the node, makes it optimal for a deviation to occur with probability q."

[10] The equilibrium is unique in the sense that any other equilibria will induce the same pricing function thus leading to exactly the same surplus for each type of sender and the market. Of course, the equilibrium could possibly be implemented by more than one set of reporting strategies.

where $z = q\frac{\theta}{1+\theta}$ and k is the unique solution to:

$$k - c = E(h|h \leq k). \tag{6}$$

The certified price is

$$p(h) = h + z - c$$

(ii) *The untruthful manager reports $r_1 = q$ when certifying and r_0 when not certifying according to a density $\varphi_0(\cdot)$*

$$\varphi_0(r) = \theta \frac{r - p_0}{p_0 - E(\pi|h < k)} \frac{f_h(r-q)}{2F_h(k)} \text{ for } r \in [p_0, k+q] \tag{7}$$

(iii) *The truthful manager with $s = -q$ does not certify. The truthful manager with $s = q$ and the untruthful manager certify if and only if:*

$$h \geq k. \tag{8}$$

Several limit cases provide a useful overview of the equilibrium's main properties. When $\theta = 0$ (the manager is entirely untruthful), then $z = E(s) = 0$ and $p_0 = E(h|h \leq k)$. This corresponds to the standard costly disclosure environment (Jovanovic 1982) where market expectations are affected only by the certification decision but not by uncertified reports. As credibility θ increases, there is some information conveyed by favorable uncertified reports, i.e., $p_0 > E(h|h \leq k)$. Put (slightly) differently, the region of uncertified reports where the manager is identified as truthful and the report fully impounded into the price is greater with more credibility. In a sense, this observation extends some of the intuitions offered by Benabou and Larocque (1992); in their model, the outcome space is binary and, as a result, if ex-ante credibility is too low, there is no possibility to convey information (Proposition 1, p. 933, and Proposition 4, p.939). In comparison, in our environment, there is always some scope to credibly report sufficienty unfavorable events and thus we find that, as long as there is some credibility, some information about unfavorable events, below p_0, will always be conveyed. Interestingly, this property appears in a special type of communication equilibria in Morgan and Stocken (2003) (which they call semi-responsive equilibria because prices tend to be responsive to the unfavorable reports). Their environment is somewhat different from ours because, as in Sobel (1985), it features uncertain incentive misalignment and, as a result of strategic reporting by both types, has many other types of equilibria. Yet, another way to think about both approaches is that equilibria where prices perfectly respond to unfavorable information would tend to be the most reasonable ones if (all other things being equal) some senders had a preference for truthfulness.

When $c \approx 0$ (the certification cost is nearly zero), the maximum price conditional on no certification becomes equal to $p_0 = z - \sigma$. Thus, the untruthful manager always certifies h, because if she did not certify, the market would assume the worst possible realization of h. Note that, as long as $\theta > 0$, untruthful managers with $s = -q$ strictly prefer not to certify and, thus, as c becomes small, the equilibrium does not converge to unravelling to complete certification. Even when c is exactly equal to zero, a certification threshold $k = q - \sigma$ is an equilibrium because managers with $s = -q$ reveal their information via the alternative informal report. Interestingly, while the no-certification equilibrium obtained earlier is compatible with other refinements (such as Cho-Kreps or D1), it is not perfect sequential because it requires a negative revision of beliefs about s conditional on a certification. If $c \to \sigma$, the market for certification fully shuts down and no firm certifies. Of course, in this case, the maximum price of an uncertified firm converges to $p_0 \to \hat{p}$.

Corollary 1 *(i) The maximum price of uncertified firms, p_0, increases in θ, c and decreases in σ. (ii) The maximum price of certified firms, $p(h)$, increases in h, q and θ. (iii) The certification threshold, k, increases in c, but decreases in σ.*

The credibility of the firm, θ, has a positive effect on the firm's maximal prices $p(h)$ and p_0. An increase in θ raises the probability that good news are true thereby reducing the discount the market applies to good news, which implies an increase in z. Following the same logic, a higher credibility raises the odds of a high report being truthful in the uncertified market, thus increasing the maximum uncertified price p_0. This effect is related to the idea that more aligned preferences may lead to more informative communication (e.g., Crawford and Sobel, 1982).[11] This occurs through a different mechanism. Communication occurs here because the receiver is uncertain about the truthfulness of the sender and the receiver simultaneously learns about truthfulness of the sender and the state of nature. As an important difference, any equilibrium in our environment features some communication (i.e., *babbling* is generically not possible) because the untruthful manager will always exploit some of this learning to her advantage,

[11] A higher θ does not imply more informative communication, in the sense of Blackwell (1951). A higher credibility θ may lead to more aggressive reporting and thus to extreme uncertainty about the firm's value ex-post when this value is very high. A higher θ is however consistent with more informative communication in the sense of integral precision (see e.g. Ganuza and Penalva, 2010.)

thus revealing some information about her type.[12]

Despite the positive effect of θ over prices, the certification threshold of the untruthful manager, k, is exactly the same as that arising when $\theta = 0$ (Jovanovic 1982, Verrecchia, 1983, 1990). This is somewhat surprising as one could expect that credibility would induce untruthful managers to substitute certification for informal communication, leading to a threshold that increases in θ. In a sense, this intuition is generally correct (in the absence of A5) and would always be present if there was no soft information (see Appendix C). However, because credibility increases both prices, in the uncertified and certified market, it may not affect the certification threshold. This being said, credibility will always reduce the likelihood of certification because, even when the threshold k is fixed, truthful managers are less inclined to certification (as discussed below.)

A higher cost of certification c has two opposing implications for the maximum price in the uncertified market p_0. First, as in standard models of certified disclosure, it induces firms with higher value to move to the uncertified market. Second, because untruthful firms are more prevalent in the certified market, such change in c also increases the likelihood of an untruthful firm in the uncertified market which, on its own, would decrease prices in the uncertified market. Yet, because (on average) certified firms have higher value than uncertified firms, we show that the first effect dominates the second, so the increase in certification costs leads to a higher p_0 despite raising the chances that the report is not truthful.

Consider next the effect of greater uncertainty relating to soft information, q. One would expect that more uncertainty should lead the market to set prices more cautiously, anticipating perhaps more misreporting by untruthful managers. However, this does not occur: maximum prices are always formed conditional on a truncated distribution for s (adjusted by a credibility factor) whose mean increases in the level of uncertainty q. This effect explains why p_0 and z both increase in the presence of more uncertainty.

Things are very different for the uncertainty relating to hard information, σ. Clearly, σ plays no role in the certified market, where h is perfectly revealed by the certification. However, whenever there is asymmetric information about h, a higher σ diminishes the

[12] This is true only generically. To see why babbling could be possible, consider the possibility that $\gamma = \frac{1}{2}$ and assume that there is only soft information. Then, if the untruthful manager used a pure reporting strategy whereby $r_1 = -s$, then $E(s|x_1) = 0$ for any report x_1. In this case, the untruthful manager would fully "jam" the information that truthful managers would like to convey. Ex-ante this equilibrium would maximize the expected welfare of truthful managers. Note that this equilibrium is a knife edge case that requires $\gamma = \frac{1}{2}$.

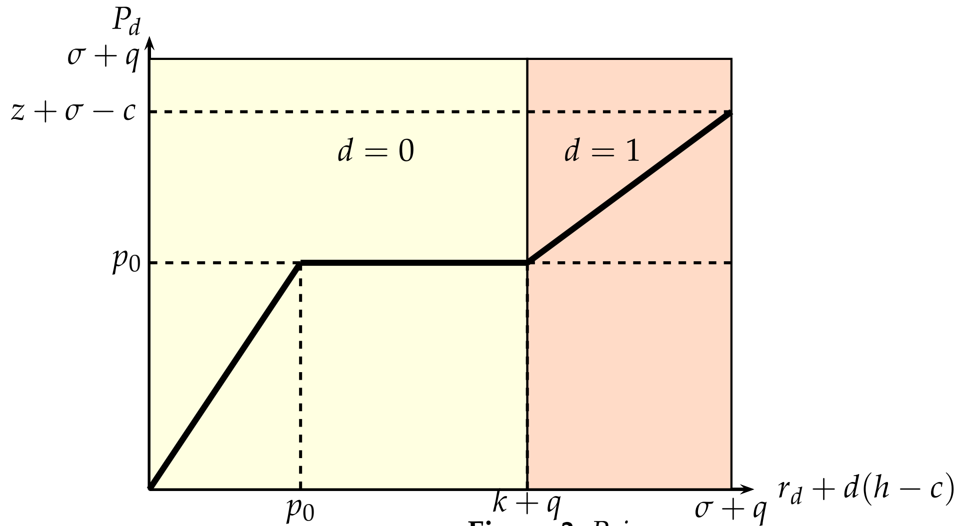

Figure 3: *Prices*

value of the firm conditional on unfavorable information about h. Since no certification is, by itself, interpreted as a negative signal about h, the value of an uncertified firm decreases in σ. This decreases p_0 as σ increases.

4.1 The likelihood of certification

In this section we examine what certification indicates about the firm's assets and the manager's truthfulness.

As previously mentioned, untruthful managers have a higher propensity to certify information because they experience stronger credibility problems and certification is their only means of overcoming the market's skepticism. By contrast, truthful managers do not need to certify h when soft information is unfavorable: in that case, the manager attains full credibility by reporting total assets truthfully. It is the manager with the greatest discretion who, by certifying, restricts that discretion the most.[13]

Interestingly, even though we have assumed away direct linkages between soft and hard information, the certification strategy of truthful managers induces a positive correlation between the value of soft assets and the decision to certify hard assets. In other words, the value of soft assets among certified firms is on average higher than among uncertified firms.

In general, the extent to which the manager relies on certification depends on a trade-off between the credibility benefits of certification and certification costs.

Corollary 2 *The probability of certification decreases in credibility, θ, decreases in the cost of certification, c, and increases in the uncertainty of hard information, σ.*

[13]Ex-ante, of course, the most affected by the possibility of certification is also the untruthful manager.

An increase in θ reduces the need to certify information by making the firm more credible ex-ante, both in the certified and uncertified market. This is natural in our model but contradicts the way certification (i.e., disclosure) is often viewed in the empirical literature, where a higher frequency of certification is interpreted as a sign of greater transparency which would (among other things) result in a lower cost of capital (see, e.g., Botosan, 1997). In our model, a higher propensity to certify information reveals greater ex-ante credibility problems (low θ), not greater transparency. The reduction in the likelihood of certification caused by an increase in θ, reveals the benefits of having more credibility. When the market believes the manager is more likely to be truthful the firm saves certification expenses thus increasing its expected value. Hence if γ was a manager's characteristic, as opposed to the firm's characteristic, then more credible managers would manager more valuable firms.

An increase in the volatility of hard information, σ, results in a higher probability of certification because more volatility lowers the price of uncertified firms, p_0, thus raising the benefits from certification. From an empirical perspective, this suggests that firms operating in riskier environments should exhibit a higher frequency of certification. Lastly, the effect of c over the firm's certification choice is straightforward. A higher c discourages certification by uniformly reducing the surplus the manager can derive from certification.

4.2 The likelihood of misreporting

A related question is how frauds (misreporting) are distributed across the certified and the uncertified markets. To answer this question, let us simply recall that untruthful managers certify more often and, therefore, misreporting must be more prevalent in the certified market. Said differently, markets (on average) perceive a higher likelihood of the manager being untruthful when certification is observed. As we show in the next corollary, the probability of certification also depends on ex-ante credibility, and the volatility of the hard and soft information.

Corollary 3 *(i) The probability of misreporting in the certified market decreases in θ.*

(ii) The probability of misreporting in the uncertified market decreases in θ and σ but increases in c.

Even though more ex-ante credibility does not affect the certification threshold k, it still reduces the demand for certification, because truthful managers are less prone to certification than untruthful ones. Conversely, the demand for certification is particularly strong when markets are skeptical about the credibility of firms.

Consider the effect of the certification cost c. Naturally, an increase in certification costs shifts firms towards the uncertified market. However this shifts is stronger among untruthful managers. To see why, note that changes in c do not affect the certification choice of a truthful manager who observes unfavorable soft information ($s = -q$), because the market always believes an informal report that soft information is unfavorable. By contrast, an increase in c always shifts some of the untruthful managers, whose hard assets h lie on the certification margin, towards the uncertified market. This self selection effect explains why the odds of misreporting by uncertified firms increase in c.

The same logic explains why the probability of misreporting in the uncertified market decreases in σ. A greater σ increases the overall probability of certification, but it does so more strongly among untruthful managers. By reducing the fraction of untruthful managers in the uncertified market, a greater volatility of hard information σ results in a lower probability of misreporting in the uncertified market.

At a more conceptual level, this set of comparative statics imply that there are situations in which untruthful types tend to crowd toward the certified market, seemingly in contrast to the standard result that better types always choose to certify (Grossman and Hart 1980, Milgrom 1981, Grossman 1981). This contradiction is only apparent: when it comes solely to hard information, the usual intuition applies and managers with higher hard information certify. The crowding of untruthful types toward the certified market occurs over soft information that cannot be certified and, even though this offsetting effect reduces the certified price, it is still the case that certified firms have on average (though not necessarily ex-post) higher value than uncertified firms.

4.3 The magnitude of misreporting

Perhaps as important as the frequency of misreporting is the magnitude of misreporting, especially given that often the most costly market regulations are triggered by the discovery of a few large-scale frauds. For example, the Sarbanes-Oxley Act of 2002 was enacted as a reaction to four or five major corporate accounting scandals (e.g., Enron, Tyco, Adelphia, Peregrine systems and Worldcom.) In this section we examine the relation between the frequency and magnitude of frauds.

Define the expected magnitude of frauds in market d as follows

$$F_d = E\left[r_d\right] - E\left[\pi|d, \tau = 0\right]. \tag{9}$$

Hence, F_d measures the average overstatement incurred by the untruthful manager in market $d \in \{0, 1\}$.

Corollary 4 *(i) The average magnitude of overstatements in the uncertified market,*

$$F_0 = q + \frac{\theta + 2}{2q} Var\left(h|h < k\right),$$

increases in θ, c and q.

(ii) The average magnitude of overstatements in the certified market,

$$F_1 = q,$$

increases in q.

We can draw three insights from this corollary. First, the magnitude of frauds is particularly large when the firm's credibility, θ, is high. What stimulates large overstatements is precisely the trust that markets assign to financial reports. But a high level of trust is possible only if the firm's propensity to misreport information is low, i.e., if the firm has a high level of credibility.

Second, the cost of certification c increases the magnitude of reports in the uncertified market without affecting the magnitude of reports in the certified market. Higher certification costs induce a first order stochastic increase in the distribution of the uncertified report r_0, thus increasing $E\left(r_0\right)$. This is due to the selection effect in the certified market. As certification becomes more expensive, some certified firms move towards the uncertified market, increasing the average value of uncertified firms. Since the market regards uncertified firms more favorably, untruthful managers start claiming larger values. Of course, this does not by itself imply that F_0 increases in c because there is a countervailing effect: the true value of untruthful firms in the uncertified market ($E\left(h|h < k\right)$) also increases in c (recall that k increases in c.) However, this is only a second order effect as compared with the effect of c on the average report of untruthful firms, $E\left(r_0\right)$.

Third, the volatility of information tends to increase the magnitude of overstatements. In particular, a higher volatility of soft information, q, increases the overstatement F_1. More generally, when s has a continuous distribution, any increase in the risk of s leads to larger overstatements. More risky soft information means that the likelihood of the tails is higher which, by increasing the credibility that s is very large, leads to more aggressive reporting.

The effect of σ on the magnitude of frauds is ambiguous: by reducing the perceived value of uncertified firms, the untruthful manager experiences incentives to be less aggressive. However, by inducing more certification, an increasing in σ decreases the actual value of untruthful firms.

5 Endogenous certification fee

We develop here an extension of the model in which certification costs are endogenous. We assume that prior to the release of the report, a monopolistic certifier publicly announces a fee c^* which the firm must pay for the certification of h. As in Lizzeri (1999), we assume that the certifier is restricted to announce a non-contingent fee, that is the fee c^* cannot depend on the realization of π.

The certifier maximizes expected profits, thus c^* is defined as

$$c^* \in \arg\max_{\hat{c}} \Pi(\hat{c}) \equiv \Pr(d = 1|\hat{c}) \cdot \hat{c},$$

so the certifier's expected demand is the probability of certification. The manager then privately observes (h, s, τ) and, as in the baseline model, chooses whether to certify and (if untruthful) what to report. For simplicity, we assume that the certifier learns the exact value of h at no cost and, if hired, truthfully reveals h to the market (unlike in Lizzeri 1999, here the certifier is not allowed to pre-commit to a noisy certification technology.)[14]

Corollary 5 *(i) There exists a unique certification equilibrium in which*

$$c^* = \arg\max_{\hat{c}} \Pr(h > k(\hat{c}, \sigma))\, \hat{c}. \tag{10}$$

(ii) The optimal fee c^* increases in σ but is independent of θ.

(iii) The probability of certification $\Pr(d = 1|c^*)$ *is independent of σ.*

[14]Lizzeri shows that the certifier would optimally commit to a completely noisy technology absent any credibility. Although revisiting this question here is beyond our current objective, this does not seem to be the case when the manager is partly credible because informal communication would substitute for a noisy certification (in Lizzeri (1999), the market assigns the worst possible belief if certification is not observed).

(iv) The certifier's profits,

$$\Pi^* = \frac{\theta/2+1}{1+\theta} \Pr\left(h > k\left(c^*, \sigma\right)\right) c^*,$$

increase in σ and decrease in θ.

The certifier's expected profits are adversely affected by the credibility of the firm but favorably affected by the volatility of hard information σ. A higher credibility θ decreases the demand for certification whereas a higher σ increases it. Note that given our distributional assumptions, the certifier's fee c^* does not depend on θ, which only has the effect of scaling the expected profits of the certifier. Also note that c^* is such that the probability of certification is independent of σ. The reason is that the certifier exploits a higher volatility σ by charging higher fees, so that in equilibrium the actual probability of certification does not vary in σ. This result contradicts a basic prediction of the disclosure literature (see e.g., Verrecchia, 1990) asserting that a greater variance of h should induce a higher probability of disclosure.[15]

6 Investment efficiency

Suppose now that, in order to generate the firm's terminal value π, the manager must incur investment K, where the investment decision is publicly observable. If the manager invests, then she could either report the value of π in the uncertified market, in which case the firm's price would depend on the credibility of her report. Alternatively, the manager could appeal to the certified market market, where the value of h would be certified at a cost c. As before, the manager's objective function is to maximize the stock price net of certification expenses. For simplicity, we assume that

$$K > \sigma - q,$$

so that investing is never (socially) optimal when soft information is unfavorable. To consider the most interesting case we also assume that θ is sufficiently high so that, p_0, as defined by Eq. (5), is such that

$$p_0 > K. \tag{11}$$

[15]The invariance of the probability of certification with respect to σ can also be demonstrated when h is normally distributed, as in Verrecchia (1990).

Under this assumption, investment is feasible even in the absence of certification. That is, in principle (if the manager's report is sufficiently high) the firm can be sold in the uncertified market at a price that exceeds the necessary investment K.

We would like to understand how the credibility of the firm θ and the certification cost c affect investment efficiency EF as represented by the firm's expected profits, net of certification expenses. We denote investment by the dummy variable $T \in \{0,1\}$ where $T = 1$ represents the case in which the manager invests and $T = 0$ denotes the case in which the manager abstains from investment. EF can be written as

$$EF = \Pr(T=1)\, E(\pi - K - cd | T = 1). \tag{12}$$

Given risk neutrality, expected returns are always zero. However, EF is indirectly related with the notion of cost of capital: financing the firm's project is infeasible when EF is negative. Put differently, bigger projects can be financed when EF is larger.

Corollary 6 *There exists a unique investment equilibrium in which*

$$T = \begin{cases} 1 & \text{if } \{\{\tau = 0\} \cup \{\tau = 1, \pi \geq K\}\} \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

In such equilibrium, EF is given by

$$EF = \frac{\theta}{1+\theta} \frac{\int_{K-q}^{\sigma} (h + q - K)\, dF_h(h)}{2} - \frac{K}{1+\theta} - \Pi. \tag{14}$$

where $\Pi = \frac{\theta/2+1}{1+\theta} c * \Pr[d = 1|c]$.

In this setting there are two sources of inefficiency. First, when $\tau = 0$, the manager always invests and sells the firm even when the project has negative NPV, because she can always overstate its NPV in the uncertified market –by contrast, under $\tau = 1$, the manager only invests when the firm's NPV is positive. The second inefficiency is related to certification expenses: the manager incurs costly certification as a means of retaining a greater share of the surplus, but from a social perspective this expense is a deadweight loss.

Corollary 7 *(i) EF increases in the cost of certification c if and only if $c \geq c^*$ where c^* is given by* (10).

(ii) If $c = c^*$, then EF decreases in σ.

An increase in the certification cost c reduces the return from investing in the certified market (recall that in the certified market, carrying out the investment requires an outlay $K + c$ as opposed to just K.) If the probability of certification were fixed, a higher c would reduce the average return to investment, thereby reducing EF. However, the increase in c generates also a countervailing effect that is particularly strong when $c \geq c^*$. Increasing c reduces the probability of certification thereby alleviating the inefficiency that arises from the manager's tendency to certify h. In fact, when $c \geq c^*$ increasing c may strongly reduce the probability of certification so that the expected certification expense, Π, diminish. Note that the expected certification expenses correspond to the certifier's expected profits, which are maximized when $c = c^*$.

Consider the effect of σ on efficiency. Again, there are two opposed effects. First, there is the classic option value effect: a greater σ increases the firm's option value. Second, there is the certification effect: a higher σ increases the propensity of the manager to certify h which in turn leads to higher certification expenses. This effect dominates the option value effect when θ is low. More surprisingly, when the certification cost is endogenous, as in Section 5, an increase in σ is always detrimental to the firm's ex-ante value.

These results have implications for the literature on disclosure and cost of capital (see e.g., Cheynel, 2010) and may explain why the related empirical evidence seems inconclusive. In this model, the association between disclosure (i.e., certification) and the efficiency EF is ambiguous and ultimately depends on the manager's credibility θ. That is, while an increase in σ generally leads to a higher probability of disclosure (as represented by the probability of certification), an increase in σ may either lead to a lower EF when θ is relatively low or to a higher EF when θ is relatively high.

7 General case

Here we generalize the model of Section 2 in two directions. First, we assume that $\pi = g(h, s)$ where $g : \mathbb{R}^2 \to \mathbb{R}$ is an increasing function of both arguments. Second, we assume that h and s are continuous random variables whose p.d.f. and c.d.f. are denoted f_l and F_l for $l \in \{h, s, \pi\}$. Furthermore, we assume that f_l is positive over $\left[\underline{l}, \bar{l}\right]$ and, as before, we normalize its mean to zero. Finally, to ensure the existence of an equilibrium with positive probability of certification we assume that $c < \bar{c}$, where the value of $\bar{c}$ is provided in the Appendix, Eq. (32).

As before, the equilibrium is characterized by three numbers: the maximum price of

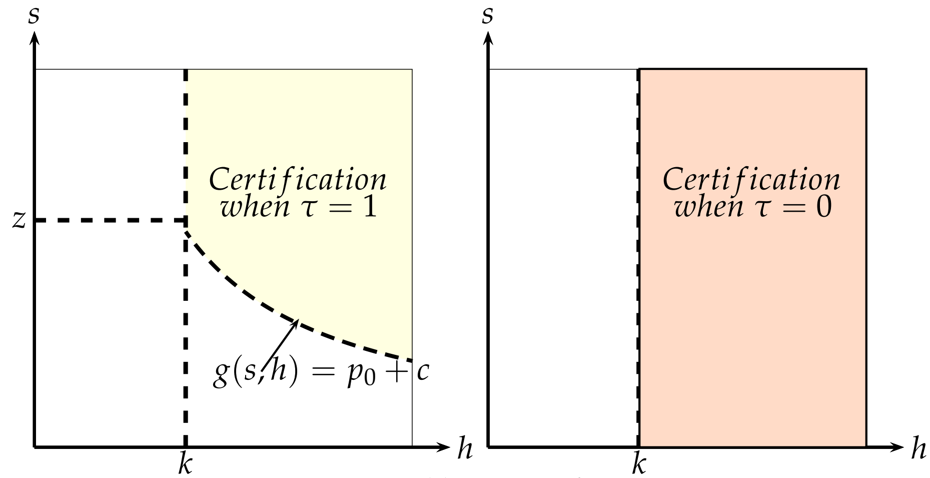

Figure 4: *Equilibrium certification*

uncertified firms p_0, the maximum market value of soft information for certified firms z, and the certification threshold of the untruthful manager k.

Proposition 5 *There is an equilibrium characterized by three numbers p_0, z and k such that*

$$\theta E\left(\pi - p_0 |\left(h,s\right) \in S_0\right) \Pr\left(\left(h,s\right) \in S_0\right) + E\left(\pi - p_0 | h < k\right) F_h\left(k\right) = 0, \tag{15}$$

$$\theta E\left(s - z | s > z\right) \Pr\left(s > z\right) = z, \tag{16}$$

and

$$p_0 = g\left(k, z\right) - c. \tag{17}$$

where the set S_0 is defined by

$$S_0 \equiv \left\{(h,s) : \left\{\pi \in (p_0, p_0 + c)\right\} \cup \left\{\pi > p + c, h < k\right\}\right\}$$

Figure 4 shows the structure of certification choices when $g\left(h,s\right) = h + s$. There we see that the untruthful manager certifies with a higher probability than the untruthful manager. The truthful manager certifies when both hard and soft information are favorable. In particular, the truthful manager certifies when $\pi \geq p_0 + c$ and $h \geq k$. We also see that certified firms are on average high value firms, yet some uncertified firms are more valuable than some certified firms. For example, firms with very favorable soft information vis-a-vis their hard information sometimes are not certified when $\tau = 1$. By contrast, firms with very unfavorable soft information relative to their hard information may be certified under discretion, $\tau = 0$.

8 Concluding remarks

A large portion of what we know about voluntary disclosure arises from two types of models: models in which the disclosure, when it occurs, is truthful but potentially costly, and models in which disclosures are unverifiable but some common interest between sender and receiver makes some communication possible. In this paper, we bridge the gap between the two disclosure forms and develop a theory of choice over voluntary disclosure alternatives. That is, our model not only speaks about what information is disclosed but also how the information is disclosed.

The model provides a variety of new implications, many of which have not been, to our knowledge, empirically tested. Specifically, we predict that: (i) conditional on certification, managers make more aggressive reports about all other assets that may not be certified, (ii) investors discount these reports less than they would have absent certification, (iii) the likelihood of a fraud is greater conditional on a certification and (iv) negatively related to the size of the fraud, (v) the likelihood of frauds in the uncertified market is decreasing in the variance of hard assets and increasing in the certification costs, (vi) the size of overstatements is increasing in the volatility of the soft information and negatively related to the likelihood of frauds, (vii) managers with frauds are more likely to have certified information, (viii) certification is less likely and frauds are larger in markets with more perceived managerial credibility.

Lastly, we point to some of the inherent limitations of our model and to (what we believe) seem interesting avenues for further research in our context. First, our approach focuses on a single period and, as any such model, is subject to the very real caveat that a forward-looking manager anticipating the consequences on any leaked information (such as propensity to be untruthful) on future periods would behave quite differently. In particular, by choosing to disclose or certify in a certain manner, the manager may acquire ex-post credibility and, thus, in future periods, achieve higher market prices. Extending the model to a dynamic setting would allow us to understand how managers build and spend reputations over time. Second, we have focused on an environment in which hard and soft information are additively separable, leaving aside questions relating to hard and soft information being complements or substitutes. This excludes reasonable situations in which, for example, the firm holds a receivable and the value of that receivable is the product of the probability of payment with the size of the receivable (complements) or when a firm with an innovative leader can achieve value even when existing assets are not in place (substitutes). Third, since new investors are entirely price-protected (and make zero net surplus), we are unable to make any statements about the desirability of disclosure forms to capital providers, in particular relative to the interest of original owners.

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A Omitted Proofs

Proof of Lemma 2. Given that the untruthful manager always reports that $\pi \geq \hat{p}$, and since the price must be constant for any report that $\pi \geq \hat{p}$, then by Bayes' rule

$$\frac{\gamma\left(1-\Pr\left(\pi<\hat{p}\right)\right)E\left(\pi|\pi>\hat{p}\right)}{\gamma\left(1-\Pr\left(\pi<\hat{p}\right)\right)+\left(1-\gamma\right)}+\frac{\left(1-\gamma\right)E\left(\pi\right)}{\gamma\left(1-\Pr\left(\pi<\hat{p}\right)\right)+\left(1-\gamma\right)}=\hat{p}.$$

Using $E\left(\pi\right)=0$, and rearranging yields

$$\hat{p}=\theta E\left(\pi-\hat{p}|\pi>\hat{p}\right)\Pr\left(\pi>\hat{p}\right). \tag{18}$$

Evaluating at $\hat{p}=0$ (resp., $\hat{p}=q$), the RHS (resp., LHS) is greater than the LHS (resp., RHS). Uniqueness is established by noting that RHS is strictly decreasing in $\hat{p}$ whereas LHS is strictly increasing. ■

Proof of Proposition 3. To obtain the manager's reporting strategy note that given $\varphi_0\left(x_0\right)$, and the fact that the price must be constant for any $x_0 \geq \hat{p}$, then by Bayes' rule

$$\frac{\gamma f_\pi\left(x_0\right)x_0+\left(1-\gamma\right)\varphi_0\left(x_0\right)E\left(\pi\right)}{\gamma f_\pi\left(x_0\right)+\left(1-\gamma\right)\varphi_0\left(x_0\right)}=\hat{p}.$$

Solving for $\varphi_0\left(x_0\right)$ gives $\varphi_0\left(x_0\right)=\theta\frac{x_0-\hat{p}}{\hat{p}}f_\pi\left(x_0\right)$. One can verify that φ_0 is a well defined p.d.f., i.e., $\int_{\hat{p}}^{\overline{\pi}}\varphi_0\left(r\right)dr=1$. ■

Consider the class of equilibria defined by the following refinement.

Assumption 6 (R) *(1) Beliefs are continuous; (2) the untruthful manager certifies with probability in (0,1); and (3) in both markets, reporting is (at least somewhere) informative.*

R(1) seems like a natural restriction: there exist equilibria where beliefs experience upward jumps (at the lower bound of the manager's reporting strategy, in the uncertified market) but in such equilibria the untruthful manager must, on average, understate the actual value of the firm –at the lower bound– so to be able to induce an upward jump on beliefs. Although this is a theoretical curiosity, it has little, if some, economic content. We conjecture that continuous equilibria maximize the ex-ante value of the firm.

R(2) rules out the posibility of an equilibrium where the untruthful manager certifies with probability one. The existence of an equilibrium in which the untruthful manager certifies with probability one crucially relies on support of f_π being disconnected.

R(3) rules out *babbling*-like equilibria, where market beliefs are everywhere independent of reports, so that prices are only affected by whether or not the manager certifies h or not. A necessary condition to sustain such equilibria is $\gamma \leq \frac{1}{2}$. Such uninformative equilibria would be refined by Farrell's (1993) neologism proofness or Grossman and Perry's (1990) PSE concept.

Under Assumption (R) we prove that the equilibrium characterized by Proposition 4 is unique.

We rely on a series of lemmas.

Lemma 7 *Given R(3), conditional on certification, and for any value of h, the untruthful manager must report that $s = q$, so that $r_1 = q$ for all h.*

Proof. R(3) implies that there is at least some value of h, say h', such that

$$E\left(\pi | d = 1, h', x_1 = q\right) > E\left(\pi | d = 1, h', x_1 = -q\right).$$

This in turn implies that given h' the untruthful manager should report $r_1 = q$. R(1) ensures that this must happen for all h that is certified in equilibrium. Otherwise, conditional on certification of h and a report $x_1 = q$, beliefs would be either $h - c$ when the reporting strategy is uninformative and by $h - c + E\left(\pi | d = 1, h, x_1 = q\right) > h - c$ when the reporting strategy is uninformative. But this would contradict the continuity of beliefs. ■

Lemma 8 *The price given certification of h is given by*

$$P\left(\{d = 1, h, x_1\}\right) = h - c + \min\left(x_1, z\right) \tag{19}$$

where $z = q\frac{\theta}{\theta+2}$.

Proof. The proof relies on the previous lemma and Bayes' rule and is omitted. ■

As usual, we adopt the convention that the manager certifies when indifferent.

Lemma 9 *Given p_0, the untruthful, and the truthful with $s = q$ certifies h if and only if $h \geq k$, where k is defined by*

$$k + z - c = p_0. \tag{20}$$

The truthful with $s = -q$ certifies if and only if

$$h - q + c \geq p_0. \tag{21}$$

Lemma 10 *Let S_0 be the support of the untruthful reporting strategy given no certification. Then S_0 must be an interval, and must be given by $[p_0, k+q]$.*

Proof. Given R(2) the set of h such that both the untruthful and untruthful does not certify has positive measure. This in turn means that S_0 must also have positive measure. If not, so that S_0 were countable, then any report on S_0 would induce a price

$$E(h|h<k) < k+q,$$

(given the certification strategy of the untruthful.) Now, for $E(h|h<k)$ to be equal to p_0 (so that it is optimal for the untruthful to reveal her type in the uncertified market) then $k+q$ should belong to S_0. However, this implies that for ε sufficiently small there is an open ball with radius ε centered around $k+q$, denoted $B_\varepsilon(k+q)$, containing a report $\dot{x}_0$, such that $\dot{x}_0$ is not certified by the truthful and not released by the untruthful. Since ε is arbitrarily small, then issuing a report $\dot{x}_0$ would lead to a price arbitrarily close to $k+q > p_0$, a contradiction. Next we show that S_0 must be an interval. Suppose not. Let $\underline{x_0}$ =inf$\{S_0\}$ and $\overline{x_0} = \sup\{S_0\}$. Then there is a report $x_0^* \in (\underline{x_0}, \overline{x_0})$ such that the untruthful manager does not release such report x_0^*. Hence the price given a report x_0^* should be given by $P\{d=0, x_0^*\} = x_0^*$. On the other hand, if the equilibrium is informative as required by R(3), the untruthful should never report the lowest value of π, hence $\underline{x_0} > \underline{\pi}$. By continuity $P(\{d=0, \underline{x_0}\}) = \underline{x_0}$, which is a contradiction (because $x_0^* > \underline{x_0}$). It is easy to see that $\overline{x_0} = k+q$. Otherwise, reporting a total value of $k+q$ would lead to a price $k+q > p_0$. Similarly, R(3) ensures that the lowest bound of S_0 be p_0, given that $\underline{x_0} > \underline{\pi}$. ■

Lemma 11 *The price given no certification and any report $x_0 \in S_0$ must equal p_0.*

These observations allows us to obtain p_0 uniquely, as we next show.

Proof of Proposition 4. By Bayes' rule, p_0 must solve:

$$\frac{\Pr(\tau=1|d=0, x_0 \geq p_0)\, E(\pi - p_0|d=0, x_0 \geq p_0, \tau=1)}{\Pr(\tau=0|d=0, x_0 \geq p_0)\, E(p_0 - \pi|d=0, x_0 \geq p_0, \tau=0)} = 1. \tag{22}$$

Define k to be the threshold that makes the untruthful manager indifferent between certification and no certification, i.e., $k+z-c = p_0$. Then,

$$\Pr(\tau=0|d=0, x_0 \geq p_0) = \frac{(1-\gamma)\, F_h(k)}{\gamma \Pr(d=0, x_0 \geq p_0|\tau=1) + (1-\gamma)\, F_h(k)}, \tag{23}$$

Also,

$$E\left(\pi | d=0, x_0 \geq p_0, \tau=0\right)=\int_{-\sigma}^{k} h \frac{f_h\left(h\right)}{F_h\left(k\right)} dh. \tag{24}$$

and

$$\Pr\left(d=0, x_0 \geq p_0 | \tau=1\right)=\gamma\left(\int_{\max(p-q,-\sigma)}^{k} \frac{1}{2} f_h\left(h\right) dh+\int_{\min(p+q,\sigma)}^{\min(p+c+q,\sigma)} \frac{1}{2} f_h\left(h\right) dh\right). \tag{25}$$

The first term arises when $s=-q$ and the second term arises when $s=q$. Plugging these expressions into (22) yields

$$\begin{aligned} & \theta \frac{1}{2} \int_{\max(p-q,-\sigma)}^{k}\left(h+q-p\right) f_h\left(h\right) dh \\ & +\theta \frac{1}{2} \int_{\min(p+q,\sigma)}^{\min(p+c+q,\sigma)}\left(h+q-p\right) f_h\left(h\right) dh \\ +\int_{-\sigma}^{\min(p+c-z,\sigma)}\left(h-p\right) f_h\left(h\right) dh \quad = \quad & 0 \end{aligned}$$

Under A5, $p_0-q<-\sigma$ and $p_0+q>\sigma$. Then p_0 must solve the simpler equation:

$$\theta \frac{1}{2} \int_{-\sigma}^{k}\left(h+q-p_0\right) f_h\left(h\right) dh+\int_{-\sigma}^{k}\left(h-p_0\right) f_h\left(h\right) dh=0. \tag{26}$$

Integrating by parts, the value of k must satisfy the following Equation:

$$\frac{\int_{-\sigma}^{k} F_h\left(t\right) dt}{F_h\left(k\right)}=c. \tag{27}$$

Since $c<\sigma$, this equation has a unique interior solution (i.e., $k>-\sigma$). For completeness, we explicitly derive the lower bound on q required in A5, $\overline{q}$, that is required for p_0 to satisfy that $p_0-q<-\sigma$ and $p_0+q>\sigma$. The value of $\overline{q}$ is given by

$$q \geq \overline{q} \equiv \max\left(\frac{(\theta+2)(\sigma-c+k)}{2}, \frac{(\theta+2)(\sigma+c-k)}{2(\theta+1)}\right). \tag{28}$$

■

The following lemma will be used in subsequent proofs.

Lemma 12 $\frac{\partial k}{\partial \sigma} \leq 0$ *and* $\frac{\partial k}{\partial c} \geq 1$. $\lim_{c \to 0} k=-\sigma$. $\lim_{c \to \sigma} k=\sigma$.

Proof. To show that $\frac{\partial k}{\partial c} \geq 1$, we apply the Implicit Function Theorem to (27).

$$\frac{\partial k}{\partial c} = \frac{1}{1 - \frac{f_k(k)}{F_k(k)} \int_{-\sigma}^{k} \frac{F_h(t)dt}{F_h(k)}}.$$

Now, the log-concavity of f_h implies that F_h and $\int_{-\sigma}^{k} F_h(t)\,dt$ are log-concave too. Thus,

$$\frac{\partial^2}{\partial k^2} \log\left(\int_{-\sigma}^{k} F_h(t)\,dt\right) = \frac{\partial}{\partial k} \frac{F_h(k)}{\int_{-\sigma}^{k} F_h(t)\,dt} \leq 0$$

hence $\frac{\partial}{\partial k} \int_{-\sigma}^{k} \frac{F_h(t)dt}{F_h(k)} \geq 0$, which in turn means that

$$\frac{\partial k}{\partial c} = \frac{1}{\frac{\partial}{\partial k} \int_{-\sigma}^{k} \frac{F_h(t)dt}{F_h(k)}} = \frac{1}{1 - \frac{f_h(k)}{F_h(k)} \int_{-\sigma}^{k} \frac{F_h(t)dt}{F_h(k)}} \geq 1.$$

■

Proof of Corollary 2. The probability of certification is given by

$$\begin{aligned}
\Pr(d=1) &= \Pr(d=1|\tau=0)(1-\gamma) + \Pr(d=1|\tau=1)\gamma \\
&= \left(\frac{1}{2}\frac{\theta}{1+\theta} + \frac{1}{1+\theta}\right) F_h(-k(c,\sigma)). \\
&= \frac{1}{2}\frac{\theta+2}{1+\theta} F_h(-k(c,\sigma)) = \frac{1}{2}\frac{\theta+2}{1+\theta} F_h(-k(c,\sigma))
\end{aligned}$$

■

Proof of Corollary 3. By Bayes' rule, the probability of misreporting in the certified market is

$$\begin{aligned}
\Pr(\tau=0|d=1) &= \frac{\Pr(d=1|\tau=0)\Pr(\tau=0)}{\Pr(d=1|\tau=0)\Pr(\tau=0) + \gamma\Pr(d=1|\tau=1)} \\
&= \frac{(1-\gamma)F_h(-k(c,\sigma))}{(1-\gamma)F_h(-k(c,\sigma)) + \gamma\frac{1}{2}F_h(-k(c,\sigma))} \\
&= \frac{1}{1+\frac{\theta}{2}}
\end{aligned}$$

In the uncertified market,

$$
\begin{aligned}
\Pr\left(\tau = 0 | d = 0\right) &= \frac{\left(1-\gamma\right) F_h\left(k\left(c,\sigma\right)\right)}{\left(1-\gamma\right) F_h\left(k\left(c,\sigma\right)\right) + \gamma\left(\frac{1}{2}F_h\left(k\left(c,\sigma\right)\right)+\frac{1}{2}\right)} \\
&= \frac{1}{1+\frac{\theta}{2}\left(1+\frac{1}{F_h\left(k\left(c,\sigma\right)\right)}\right)}
\end{aligned}
$$

■

Proof of Corollary 4. The mean overstatement in the uncertified market is given by

$$
\begin{aligned}
F_0 &= E\left[r_0\right] - E\left[\pi | h < k\right] \\
&= \int_{-\sigma}^{k+q} r\varphi_0\left(r\right) dr - E\left[h | h < k\right] \\
&= \int_{-\sigma}^{k+q} r\varphi_0\left(r\right) dr - \left(k - c\right) \\
&= q + \int_{-\sigma}^{k} r\theta \frac{r+q-p_0}{z}\frac{1}{2}\frac{f_h\left(r\right)}{F_h\left(k\right)} dr - k + c \\
&= q + c - k + \int_{-\sigma}^{k} r\theta \frac{r+q-k+c-z}{z}\frac{1}{2}\frac{f_h\left(r\right)}{F_h\left(k\right)} dr \\
&= q + c - k + \frac{\theta+2}{2q}\int_{-\sigma}^{k} r\frac{\left(r+q-k+c-z\right)}{1}\frac{f_h\left(r\right)}{F_h\left(k\right)} dr \\
&= q + c - k + \frac{\theta+2}{2q}\left(\int_{-\sigma}^{k} r^2\frac{f_h\left(r\right)}{F_h\left(k\right)} dr + \left(q-k+c-z\right)\left(k-c\right)\right) \\
&= q + c - k + \frac{\theta+2}{2q}\left(\int_{-\sigma}^{k} r^2\frac{f_h\left(r\right)}{F_h\left(k\right)} dr - \left(k-c\right)^2\right) + \frac{\theta+2}{2q}\left(q-z\right)\left(k-c\right) \\
&= q + c - k + \frac{\theta+2}{2q}\left(\int_{-\sigma}^{k} r^2\frac{f_h\left(r\right)}{F_h\left(k\right)} dr - \left(k-c\right)^2\right) - \left(k-c\right) \\
&= q + \frac{\theta+2}{2q} Var\left(h | h < k\right).
\end{aligned}
$$

where it becomes apparent that F_0 increases in θ. Furthermore, log-concavity of f_h ensures that $Var\left(h|h<k\right)$ increases in k (see e.g. Heckman and Honore, 1990). Thus F_0 must increase in c. To obtain the effect of q, note that

$$
\frac{\partial F_0}{\partial q} = 1 - \frac{\theta+2}{2q^2} Var\left(h | h < k\right).
$$

which clearly increases in q. By A.5, $q \geq q = \frac{(\theta+2)(\sigma+m)}{2}$, where

$$m = E\left(h|h < k\right) = k - c.$$

Thus, evaluating $\frac{\partial F_0}{\partial q}$ at $q = \frac{(\theta+2)(\sigma+m)}{2}$yields

$$\frac{\partial F_0}{\partial q} = 1 - \frac{2}{\theta+2}\frac{Var\left(h|h<k\right)}{\left(\sigma+m\right)^2} > 0.$$

On the other hand, the average magnitude of overstatement in the certified market is simply

$$F_1 = q.$$

■

Proof of Corollary 5. The uniqueness of c^* is implied by the log-concavity of $F_h\left(\cdot\right)$. Note that

$$\max_c \Pi \equiv \max_k \frac{\theta/2+1}{1+\theta}\left[1 - F_h\left(k\right)\right]\frac{\int_{-\sigma}^{k} F_h\left(t\right)dt}{F_h\left(k\right)}$$

Since, by choosing c, the certifier indirectly determines the threshold k, one can think of the certifier as choosing k rather than c. Now the log-concavity of F_h implies that $\frac{\int_{-\sigma}^{k} F_h(t)dt}{F_h(k)}$ is also log-concave. On the other hand,

$[1 - F_h]$ must be log-convex, thus it is not clear whether $\log \Pi$ is concave in k. However, a corner solution can never be optimal. So we are left with the possibility of multiple interior solutions. Now, maximizing $\log \Pi$ yields the first order condition

$$\frac{F_h\left(k^*\right)}{\int_{-\sigma}^{k^*} F_h\left(t\right)dt} = \frac{f_h\left(k^*\right)}{1 - F_h\left(k^*\right)}. \tag{29}$$

where $k^* = k\left(c^*, \sigma\right)$. The right hand side is increasing, since the hazard rate of log-concave distributions is increasing (see e.g., Bagnoli & Bergstrom, 2005). By contrast, the left hand side is decreasing, by the log-concavity of f_h (see Burdett, 1996). Thus there can only be one solution to Eq. (29). ■

Proof of Corollary 7. The effect of c is implied by Corollary 5. Consider now the effect of σ. Assume now that c is endogenous, so that $c = c^*$. Then by the envelope theorem:

$$\begin{aligned}\frac{\partial EF}{\partial \sigma} &= \frac{\gamma}{2}\int_{\frac{K-q}{\sigma}}^{1} tf(t)\,dt - \left(\frac{\gamma}{2} + (1-\gamma)\right)(1 - F(w^*))\frac{\int_{-1}^{w^*} F(t)\,dt}{F(w^*)} \\ &= \frac{\gamma}{2}\left(H\left(\frac{K-q}{\sigma}\right) - \Gamma(w^*)\right) - (1-\gamma)\,\Gamma(w^*)\end{aligned}$$

where $w^* = \frac{k^*}{\sigma}$,

$$L(w^*) = (1 - F(w^*))\frac{\int_{-1}^{w^*} F(t)\,dt}{F(w^*)},$$

and

$$H\left(\frac{K-q}{\sigma}\right) = \int_{\frac{K-q}{\sigma}}^{1} tf(t)\,dt.$$

Now both $\Gamma(\cdot)$ and $H(\cdot)$ are single-peaked functions. It is easy to see that $H(\cdot)$ is maximized at zero. So $H\left(\frac{K-q}{\sigma}\right) \leq H(0)$. Moreover,

$$L(0) = \Gamma(0).$$

Finally, by revealed preferences we know that

$$L(r^*) \geq \Gamma(0) = H(0),$$

hence, when $c = c^*$,

$$\frac{\partial EF}{\partial \sigma} \leq 0.$$

■

Proof of Proposition 5. The value of z is given by

$$\theta E(s - z | s > z)\Pr(s > z) = z \tag{30}$$

which represents the maximum price of soft information that prevails when h is certified. The existence and uniqueness of z can be easily established.

To show the existence of p_0 we first define $\hat{p}$ as

$$\theta E(\pi - \hat{p} | \pi > \hat{p})\Pr(\pi > \hat{p}) = \hat{p} \tag{31}$$

which represents the maximum price that would prevail if certification was not available. Now we can define a bound for the certification cost c. We will assume that g is such that there is a number $\overline{c} > 0$, defined by

$$\hat{p} = g\left(\overline{h}, z\right) - \overline{c}. \tag{32}$$

Setting $c < \overline{c}$ ensures that a positive probability of certification is feasible in equilibrium. Finally, we define a set $S_0(x)$ as follows.

$$S_0(x) = \{(x,y) : \{g(x,y) \in [x, x+c]\} \cup \{g(x,y) > x+c, h < k(x)\}\} \tag{33}$$

where the function $k = k(x)$ is given by

$$x = g(k,z) - c. \tag{34}$$

The set $S_0(x)$ will represent the support of the manager's reporting strategy in the uncertified market when the maximum price in that market is x. We can now establish the existence of p_0, which given z, determines the value of k. First note that the function

$$\Delta_0(p,z) = \theta E(\pi - p|(h,s) \in S_0(p)) \Pr((h,s) \in S_0(p)) + E(\pi - p|h < k) F_h(k)$$

is continuous on its first argument over $[\max(p_-, \underline{\pi}), p^+]$, where

$$p_- = g(\underline{h}, z) - c,$$

and

$$p^+ = g\left(\overline{h}, z\right) - c.$$

Assume that $p_- \geq \underline{\pi}$ (the other case is trivial). Observe that $\Delta_0(x,z)$ is proportional to the rents the market obtains from paying x to an uncertified firm claiming to have a value equal to or greater than x when x is the maximum price paid in that market. Of course, given the competitive nature of the market, an equilibrium exists if there is p_0 satisfying $\Delta(p_0, z) = 0$. Now, it is easy to see that $\Delta_0(p_-, z) \geq 0$. In fact, observe that if p_- was the maximum price paid in the uncertified market, then only under $\tau = 1$ the manager would both claim a value greater than p_- and choose not to certify h. Thus the market would necessarily make positive profits at that price. By contrast, p^+ is the price of uncertified firms that would shut down certification given certification cost c. On the other hand, when no one certifies and the maximum price is given by p^+, then the market rents, conditional on a firm claiming a value greater than p^+ would be given by

$$\Delta_0\left(p^{+},z\right)=\Gamma\left(p^{+},\theta\right)$$

where

$$\Gamma\left(p^{+}\right)\equiv\theta E\left(\pi-p^{+}|\pi>p^{+}\right)\Pr\left(\pi>p^{+}\right)-p^{+}.$$

One can easily show that $\Gamma(\cdot)$ is a decreasing function. Furthermore, by the definition of $\hat{p}$ (see Eq. (31)) we know that $\Gamma(\hat{p})=0$. Since, by assumption, $c<\overline{c}$ (as defined by Eq. (32)) then it is clear that

$$p^{+}=g\left(\overline{h},z\right)-c\geq g\left(\overline{h},z\right)-\overline{c}=\hat{p}.$$

Hence

$$\Delta_0\left(p^{+},z\right)=\Gamma\left(p^{+},\theta\right)\leq 0.$$

Therefore, by the Intermediate Value Theorem, there must be a $p_0\in\left[p_{-},p^{+}\right]$ such that $\Delta_0\left(p_0,z\right)=0$. ■

B Outside A.5

Here we consider the case where

$$\begin{aligned} p_0+q &> \sigma \\ p_0-q &< -\sigma \end{aligned}$$

where p_0 solves the following equation

$$\Delta_0=\frac{\theta}{2}\int_{p_0-q}^{k}\left(h+q-p_0\right)dF_h+\int_{-\sigma}^{k}\left(h-p_0\right)dF_h=0 \tag{35}$$

or

$$cF_h\left(k\right)=\int_{-\sigma}^{k}F_h\left(t\right)dt-\frac{\theta}{\theta+2}\int_{-\sigma}^{p_0-q}F_h\left(t\right)dt \tag{36}$$

where

$$k=p_0+c-z.$$

Corollary 8 *There is a unique p_0 and p_0 increases in θ, and c and q.*

Proof. To show uniqueness note that

$$\frac{\partial \Delta}{\partial p_0} = \frac{\theta+2}{2}\left(\lambda\left(k\right)\left(\int_{-\sigma}^{k} F_h\left(t\right)dt - \frac{\theta}{\theta+2}\int_{-\sigma}^{p-q} F_h\left(t\right)dt\right) - F_h\left(k\right)\right)$$

where $\lambda\left(k\right) = \frac{f_h(k)}{F_h(k)}$ is the inverse hazard rate of F_h. λ is decreasing because f_h is log-concave. Therefore

$$\begin{aligned}
\frac{\partial \Delta}{\partial p_0} &\leq \frac{\theta+2}{2}\left(F_h\left(k\right) - \frac{\theta}{\theta+2}\lambda\left(k\right)\int_{-\sigma}^{p_0-q} F_h\left(t\right)dt - F_h\left(k\right)\right) \\
&= -\frac{\theta}{2}\lambda\left(k\right)\int_{-\sigma}^{p_0-q} F_h\left(t\right)dt < 0.
\end{aligned}$$

To show that p_0 increases in θ, by the Implicit Function Theorem we just need to show that $\Delta_\theta \geq 0$.

$$\begin{aligned}
\Delta_\theta &= \frac{1}{2}\int_{p-q}^{k}\left(h+q-p\right)dF_h + \frac{\theta}{2}\left(c-z+q\right)f_h\left(k\right)z' + \left(c-z\right)f_h\left(k\right)z' \\
&= \frac{1}{2}\int_{p-q}^{k}\left(h+q-p\right)dF_h + \left(\frac{\theta}{2}\left(c-z+q\right)+\left(c-z\right)\right)f_h\left(k\right)z' \\
&= \frac{1}{2}\int_{p-q}^{k}\left(h+q-p\right)dF_h + \frac{\theta+2}{2}cf_h\left(k\right)q\frac{2}{\left(\theta+2\right)^2} \\
&= \frac{1}{2}\int_{p-q}^{k}\left(h+q-p\right)dF_h + \frac{cf_h\left(k\right)q}{\left(\theta+2\right)} > 0
\end{aligned}$$

To show that p_0 increases in c note that

$$\Delta = \frac{\theta}{2}\int_{p-q}^{k}\left(h+q-p\right)dF_h + \int_{-\sigma}^{k}\left(h-p\right)dF_h$$

Thus

$$\begin{aligned}
\Delta_c &= \left(\frac{\theta}{2}\left(c-z+q\right)+\left(c-z\right)\right)f_h\left(k\right)\frac{\partial k}{\partial c} \\
&= \frac{\theta+2}{2}cf_h\left(k\right) > 0.
\end{aligned}$$

The case of q is obvious. ■

Corollary 9 *k increases in c, θ and q.*

Proof. To show this note that

$$\begin{aligned}\Delta &= \frac{\theta}{2}(c-z+q)F_h(k)+(c-z)F(k)-\frac{\theta}{2}\int_{p-q}^{k}F_h(t)\,dt-\int_{-\sigma}^{k}F_h(t)\,dt\\ &= \frac{\theta+2}{2}cF_h(k)-\frac{\theta}{2}\int_{p-q}^{k}F_h(t)\,dt-\int_{-\sigma}^{k}F_h(t)\,dt\end{aligned}$$

Then for fixed k,

$$\begin{aligned}\Delta_\theta &= \frac{1}{2}cF_h(k)-\frac{1}{2}\int_{p-q}^{k}F_h(t)\,dt\\ &\propto cF_h(k)-\int_{p-q}^{k}F_h(t)\,dt\end{aligned}$$

Using 36 one gets

$$\begin{aligned}\Delta_\theta &\propto \int_{-\sigma}^{k}F_h(t)\,dt-\frac{\theta}{\theta+2}\int_{-\sigma}^{p-q}F_h(t)\,dt-\int_{p-q}^{k}F_h(t)\,dt\\ &> \int_{-\sigma}^{k}F(t)\,dt-\int_{-\sigma}^{p-q}F(t)\,dt-\int_{p-q}^{k}F(t)\,dt\\ &= \int_{p-q}^{k}F(t)\,dt-\int_{p-q}^{k}F(t)\,dt\\ &= 0\end{aligned}$$

The effect of c is straightforward from 35. ∎

C When $q=0$

When there is only hard information, the price given certification is

$$P(\{d=1,h\})=h-c.$$

Denote by p_0 the maximum price given no certification and by k the certification threshold. Then k must satisfy

$$k-c=p_0.$$

For p_0 to be consistent with Bayes' rule, it must satisfy

$$\theta\int_{p_0}^{p_0+c}(h-p_0)\,dF_h(h)+\int_{-\sigma}^{p_0+c}(h-p_0)\,dF_h(h)=0 \tag{37}$$

Corollary 10 *Equation* (37) *has a unique solution.*

Proof. (We thank TJ Liu Tingjun for providing this proof.) Existence follows from the Intermediate Value Theorem. The proof of uniqueness relies on a series of observations. Rewrite Eq. (37) as

$$(\theta+1)\int_{p_0}^{p_0+c}(h-p_0)\,dF_h(h)-\int_{-\sigma}^{p}(p_0-h)\,dF_h(h)=0.$$

Now let

$$Q_1(p)=(\theta+1)\int_{p}^{p+c}(h-p)\,dF_h(h)$$

and

$$Q_2(p)=\int_{-\sigma}^{p}(p_0-h)\,dF_h(h).$$

thus

$$Q_1(p)-Q_2(p)=0$$

We proceed by contradiction. Suppose there exists equilibrium prices p^1, p^2 with $p^2 > p^1$ that solve Eq. (37), both leading to interior levels of certification. Note that $Q_1(p)$ and $Q_2(p)$ are both positive. Changing variables we see that

$$\begin{aligned}
Q_1(p_2) &= \int_0^c y f_h(y+p_2)\,dy \\
Q_1(p_2) &= \int_0^c y\frac{f_h(y+p_2)}{f_h(y+p_1)}f_h(y+p_1)\,dy
\end{aligned}$$

However, by the log concavity of $f_h(\cdot)$, for any $y \geq 0$

$$\frac{f_h(y+p_2)}{f_h(y+p_1)} \leq \frac{f_h(p_2)}{f_h(p_1)}.$$

Hence

$$\begin{aligned}
Q_1(p_2) &\leq \frac{f_h(p_2)}{f_h(p_1)}Q_1(p_1) \\
\frac{Q_1(p_2)}{Q_1(p_1)} &\leq \frac{f_h(p_2)}{f_h(p_1)}.
\end{aligned}$$

Similarly, one can show that

$$\frac{Q_2(p_2)}{Q_1(p_2)} > \frac{f_h(p_2)}{f_h(p_1)}.$$

which yields the contradiction. ■

Corollary 11 *p_0 is increasing in θ and c.*

Proof. The proof follows from the Implicit Function Theorem given that the LHS of Eq. (37) decreases in p_0 but increases in both θ and c. ■

Corollary 12 *k is increasing in θ and c.*