Chapter 5
FX Derivatives

B. FX Options

Options: Brief Review

Terminology

Major types of option contracts:
- *calls* give the holder the right to buy the underlying asset
- *puts* give the holder the right to sell the underlying asset.

Terms of an option must specify:
- *Exercise* or *strike price* (**X**): Price at which the right is "exercised."
- *Expiration date* (T): Date when the right expires.
- *Type*. When the option can be exercised: Anytime (*American*)
  At expiration (*European*).

The right to buy/sell an asset has a price: The *premium*, paid upfront.

More terminology:

• An option is:

- In-the-money (ITM) if, today, we would exercise it.
  For a call: **X** < S_t
  For a put: S_t < **X**

- At-the-money (ATM) if, today, we would be indifferent to exercise it.
  For a call: **X** = S_t
  For a put: S_t = **X**

In practice, you never exercise an ATM option, since there are (small) costs associated with exercising an option.

- Out-of-the-money (OTM) if, today, we would not exercise it.
  For a call: **X** > S_t
  For a put: S_t > **X**

The Black-Scholes Formula

• Options are priced based on the Black-Scholes formula. For a call option on a stock, whose price is S_t:

$$C_t = S_t\, N(d1) - X\, e^{-i*(T-t)} N(d2)$$

where

T: time to maturity,

X: strike price,

σ: stock price volatility, &

$$N(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$$

d1 = $[\ln(S_t/\mathrm{X}) + (i + \sigma^2/2)\,(T-t)]/(\sigma\sqrt{T-t})$,

d2 = $[\ln(S_t/\mathrm{X}) + (i - \sigma^2/2)\,(T-t)]/(\sigma\sqrt{T-t})$ = d1 – $\sigma\sqrt{T-t}$.

• Black and Scholes (1973) changed the financial world by introducing their Option Pricing Model. Many applications.

The Black-Scholes Formula

• Almost all financial securities have some characteristics of financial options, the Black-Scholes model can be widely applied.

• The variation for a call FX option is given by:

$$C_t = e^{-i_f*(T-t)}\, S_t\, N(d1) - X\, e^{-i_d*(T-t)} N(d2)$$

• The Black-Scholes formula is derived from a set of assumptions:
  - Risk-neutrality.
  - Perfect markets (no transactions costs, divisibility, etc.).
  - Log-normal distribution with constant moments.
  - Constant risk-free rate.
  - Costless to short assets.
  - Continuous pricing.

• The Black–Scholes model does not fit the data. In general:
  - Overvalues deep OTM calls & undervalue deep ITM calls.
  - Misprices options that involve high-dividend stocks.

• The Black-Scholes formula is taken as a useful approximation.

• Limitations of the Black-Scholes Model
  - Trading is not cost-less: *Liquidity risk* (difficult to hedge)
  - No continuous trading: *Gap risk* (can be hedged)
  - Log-normal distribution: Not realistic (& cause of next limitations).
  - Underestimation of extreme moves: *Left tail risk* (can be hedged)
  - Constant moments: *Volatility risk* (can be hedged)

Trading in Currency Options

- **Markets for Foreign Currency options**

(1) Interbank (OTC) market centered in London, NY, & Tokyo.
OTC options are tailor-made as to amount, maturity, and exercise price.

(2) Exchange-based markets centered in Philadelphia (PHLX, now NASDAQ), NY (ISE, now Eurex) and Chicago (CME Group).

- PHLX options are on spot amounts of 10,000 units of FC (MXN 100K, SEK 100K, JPY 1M).
- PHLX maturities: 1, 3, 6, and 12 months.
- PHLX expiration dates: March, June, Sept, Dec, plus 2 spot months.
- Exercise price of an option at the PHLX or CME is stated as the price in USD cents of a unit of foreign currency.

OPTIONS

PHILADELPHIA EXCHANGE

	Calls		Puts		
	Vol.	Last	Vol.	Last	
Euro				135.54	← S_t = 1.3554 USD/EUR
10,000 Euro-cents per unit.					← Size
132 Feb	...	0.01	3	0.38	
132 Mar	3	2.74	90	0.15	
134 Feb	3	1.90	...	...	← P_{call} = USD .019
134 Mar	...	0.01	25	1.70	
136 Mar	8	1.85	12	2.83	
138 Feb	75	0.43	...	0.01	
142 Mar	1	0.08	1	7.81	← P_{put} = USD .0781

X=Strike (↗ 142) T=Expiration (↖ Mar)

OPTIONS

PHILADELPHIA EXCHANGE

	Calls Vol.	Calls Last		Puts Vol.	Puts Last	
Euro					135.54 ← S_t = 1.3554 USD/EUR	
10,000 Euro-cents per unit.						
132 Feb	...	0.01	ITM	3	0.38	OTM
132 Mar	3	2.74	ITM	90	0.15	OTM
134 Feb	3	1.90	ITM	...	...	OTM
134 Mar	...	0.01	ITM	25	1.70	OTM
136 Mar	8	1.85	OTM	12	2.83	ITM
138 Feb	75	0.43	OTM	...	0.01	ITM
142 Mar	1	0.08	OTM	1	7.81	ITM

• **Note on the value of Options**

For the same maturity (*T*), we should have:

value of ITM options > value of ATM options > value of OTM options

• ITM options are more expensive, the more ITM they are.

Example: Suppose S_t = **1.3554 USD/EUR**. We have two ITM Mar puts:

X_{put} = 1.36 USD/EUR

X_{put} = 1.42 USD/EUR.

premium (X = 1.36) = USD 0.0170

premium (X = 1.42) = **USD 0.0781**. ¶

Using Currency Options

• Iris Oil Inc. will transfer **CAD 300 million** to its USD account in 90 days (UP: long **CAD 300M**). To avoid FX risk, Iris Oil decides to use a USD/CAD option contract.

Data: S_t = .8451 USD/CAD

Available Options for the following T = 90-day period:

X	Calls	Puts
.82 USD/CAD	----	0.21
.84 USD/CAD	1.58	**0.68** ← P_{put}= **USD 0.0068**
.88 USD/CAD	0.23	----

Iris Oil selects **.84 USD/CAD** put:

Cost = **CAD 300 M** * **USD 0.0068**/CAD = **USD 2.04 M**

• Iris Oil decides to use the **X = .84 USD/CAD** put ⇒ Cost: **USD 2.04M.**

At T = t+90, there will be two scenarios:

Option is ITM (exercised –i.e., S_t < X=0.84)

Option is OTM (not exercised)

Position	**Initial CF**	S_{t+90} **< .84 USD/CAD**	S_{t+90} **≥ .84 USD/CAD**
Option (HP)	**USD 2.04M**	**(.84** – S_{t+90}) * **CAD 300M**	0
Underlying (UP)	0	S_{t+90} * **CAD 300M**	S_{t+90} * **CAD 300M**
Total CF	**USD 2.04M**	USD 252M	S_{t+90} * CAD 300M

Net CF in 90 days:

USD 252M – **USD 2.04M** = USD 249.96M for S_{t+90} < .84 USD/CAD

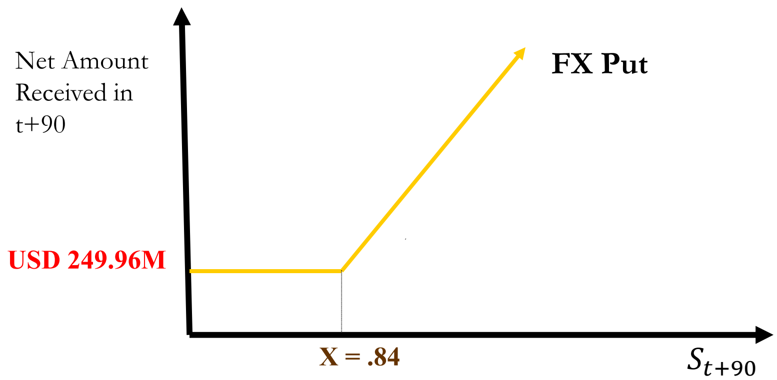
S_{t+90} * **CAD 300M** – **USD 2.04M** for S_{t+90} ≥ .84 USD/CAD

Worst case scenario (floor): USD 249.96M (when put is exercised.)

Remark: The final CFs depend on S_{t+90}!

The payoff diagram shows that the FX option limits FX risk, Iris Oil has established a floor: **USD 249.96M**.

But, FX options, unlike Futures/forwards, have an upside ⇒ At time t, the final outcome is unknown. There is still (some) uncertainty!



• With options, there is a choice of strike prices (& premiums). A feature not available in forward/futures.

• Suppose, Iris Oil also considers the **X = .82** put ⇒ Cost: **USD 0.63M**

Again, at T = t+90, we will have two scenarios:

Position	**Initial CF**	S_{t+90} **< .82 USD/CAD**	S_{t+90} **≥ .82 USD/CAD**
Option (HP)	**USD 0.63M**	(.82 – S_{t+90}) * **CAD 300M**	0
Underlying (UP)	0	S_{t+90} * **CAD 300M**	S_{t+90} * **CAD 300M**
Total CF	**USD 0.63M**	USD 246M	S_{t+90} * CAD 300M

Net CF in 90 days:

USD 246M – **USD 0.63M** = **USD 245.37M** for S_{t+90} < .82 USD/CAD

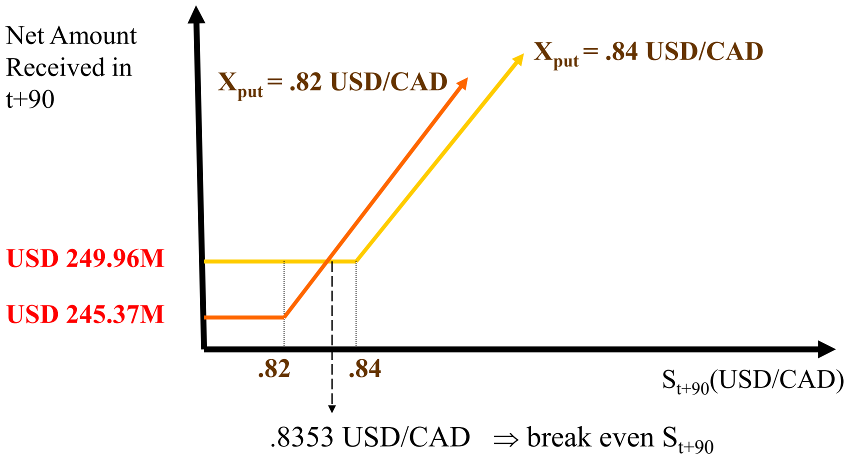
S_{t+90} * CAD 300M – **USD 0.63M** for S_{t+90} ≥ .82 USD/CAD

Worst case scenario (floor): **USD 245.37M** (when put is exercised).

• Both FX options limit Iris Oil FX risk:

- **X_{put}= .84 USD/CAD** ⇒ floor: **USD 249.96M** (cost: **USD 2.04M**)
- **X_{put}= .82 USD/CAD** ⇒ floor: **USD 245.37M** (cost: **USD 0.63M**)

Note: Higher premium, higher floor (better coverage).



Hedging with FX Options

• Hedging with Options is Simple

Situation 1: Underlying position: long in foreign currency.
Hedging position: long in foreign currency ***puts***.

Situation 2: Underlying position: short in foreign currency.
Hedging position: long in foreign currency ***calls***.

OP = underlying position (UP) + hedging position (HP-options)

Value of OP = Value of UP + Value of HP + Transactions Costs (TC)

Profit from OP = ΔUP + Δ HP-options + TC

• Advantage of options over futures:
⇒ Options simply expire if S_t moves in a beneficial way.

• Price of the asymmetric advantage of options: The TC (insurance cost).

• We will present a simple example, where the size of the hedging position is equal to the hedging options ("Naïve" or "Basic Approach").

Example: A U.S. investor is long **GBP 1 million**.
She hedges using Dec put options with **X = USD 1.60** (ATM).

Underlying position: $\mathbf{V_0}$ = **GBP 1,000,000**.
$\mathbf{S_{t=0}}$ **= 1.60 USD/GBP.**
Size of the PHLX contract: GBP 10,000.
X = USD 1.60
$P_{t=0}$ = premium of Dec put = USD .05.

TC = Cost of Dec puts = **1,000,000** * USD .05 = **USD 50,000**.
Number of contracts = **GBP 1,000,000**/ GBP 10,000 = 100 contracts.

On December $\mathbf{S_{T=Dec}}$ = **1.50 USD/GBP** ⇒ option is exercised (ITM put)

ΔUP = $\mathbf{V_0}$ * ($\mathbf{S_T}$ – $\mathbf{S_0}$) = **GBP 1M** *(**1.50** – **1.60**) USD/GBP = - USD 0.1M.
ΔHP = $\mathbf{V_0}$ * (**X** – $\mathbf{S_T}$) = **GBP 1M** * (**1.60** – **1.50**) USD/GBP = USD 0.1M.
ΔOP = -USD 100,000 + USD 100,000 – **USD 50,000** = -USD 50,000. ¶

Example:
If at T, $S_{T=Dec}$ = **1.80 USD/GBP** ⇒ option is not exercised (OTM put).

ΔUP = V_0 * (S_T – S_0) = **GBP 1M** * (**1.80** – **1.60**) USD/GBP = USD 0.2M
ΔHP = 0 (No exercise)
ΔOP = USD 200,000 - **USD 50,000** = USD 150,000. ¶

The price of this asymmetry is the premium: USD 50,000 (a sunk cost!).

FX Options: Hedging Strategies

• Hedging strategies with options can be more sophisticated:
⇒ Investors can play with several exercise prices with options only.

Example: Hedgers can use:
- Out-of-the-money (least expensive)
- At-the-money (expensive)
- In-the-money options (most expensive)

• Same *trade-off* of car insurance:
- Low premium (high deductible)/low floor or high cap: *Cheap*
- High premium (low deductible)/high floor or low cap: *Expensive*

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134 Mar	...	0.01	25	1.70	OTM; P_{put} = USD .0170
136 Mar	8	1.85	12	2.83	ITM; P_{put} = USD .0283
138 Feb	75	0.43	...	0.01	ITM
142 Mar	1	0.08	1	7.81	ITM
Swedish Krona				**15.37**	
100,000 Swedish Krona -cents per unit.					

Example: It is February 2, 2011.
UP = Long bond position **EUR 1,000,000**.
HP = EUR Mar put options: **X = 134** and **X = 136**.
S_t = 1.3554 USD/EUR.

(A) Out-of-the-money Mar 134 put.
Total cost = USD .0170 * **1,000,000** = **USD 17,000**
Floor = **1.34 USD/EUR** * **EUR 1,000,000** = USD 1,340,000.
Net Floor = USD 1.34M – **USD .017M** = **USD 1.323M**

(B) In-the-money Mar 136 put.
Total cost = USD .0283 * **1,000,000** = **USD 28,300**
Floor = **1.36 USD/EUR** * **EUR 1,000,000** = USD 1,360,000
Net Floor = USD 1.36M – **USD .0283M** = **USD 1.3317M**

• As usual with options, under both instruments there is some uncertainty about the final cash flows. ¶

• Both FX options limit FX risk:

- **X_{put} = 1.34 USD/EUR** ⇒ floor: **USD 1.323M** (cost: USD .017 M)
- **X_{put} = 1.36 USD/EUR** ⇒ floor: **USD 1.3317M** (cost: USD .0283M)

Typical trade-off: A higher minimum (net floor) amount for UP (**USD 1.3317M**) is achieved by paying a higher premium (**USD 28,300**).