CHAPTER V

FORECASTING EXCHANGE RATES

One of the goals of studying the behavior of exchange rates is to be able to forecast exchange rates. Chapters III and IV introduced the main theories used to explain the movement of exchange rates. These theories fail to provide a good approximation to the behavior of exchange rates. Forecasting exchange rates, therefore, seems to be a difficult task.

This chapter analyzes and evaluates the different methods used to forecast exchange rates. This chapter closes with a discussion of exchange rate volatility.

I. Forecasting Exchange Rates

International transactions are usually settled in the near future. Exchange rate forecasts are necessary to evaluate the foreign denominated cash flows involved in international transactions. Thus, exchange rate forecasting is very important to evaluate the benefits and risks attached to the international business environment.

A forecast represents an expectation about a future value or values of a variable. The expectation is constructed using an information set selected by the forecaster. Based on the information set used by the forecaster, there are two pure approaches to forecasting foreign exchange rates:

(1) The fundamental approach.

(2) The technical approach.

1.A <u>Fundamental Approach</u>

The *fundamental approach* is based on a wide range of data regarded as *fundamental* economic variables that determine exchange rates. These fundamental economic variables are taken from economic models. Usually included variables are GNP, consumption, trade balance, inflation rates, interest rates, unemployment, productivity indexes, etc. In general, the fundamental forecast is based on structural (equilibrium) models. These structural models are then modified to take into account statistical characteristics of the data and the experience of the forecasters. It is a mixture of art and science. (See Appendix IV.)

Practitioners use structural model to generate equilibrium exchange rates. The equilibrium exchange rates can be used for projections or to generate trading signals. A trading signal can be generated every time there is a significant difference between the model-based expected or forecasted exchange rate and the exchange rate observed in the market. If there is a significant difference between the expected foreign exchange rate and the actual rate, the practitioner should decide if the difference is due to a mispricing or a heightened risk premium. If the practitioner decides the difference is due to mispricing, then a buy or sell signal is generated.

1.A.1 Fundamental Approach: Forecasting at Work

The fundamental approach starts with a model, which produces a forecasting equation. This model can be based on theory, say PPP, a combination of theories or on the ad-hoc experience of a practitioner. Based on this first step, a forecaster collects data to estimate the forecasting equation. The estimated forecasting equation will be evaluated using different statistics or measures. If the forecaster is happy with the model, she will move to the next step, the generation of forecasts. The final step is the evaluation of the forecast.

As mentioned above, a forecast represents an expectation about a future value or values of a variable. In this chapter, we will forecast a future value of the exchange rate, S_{t+T} . The expectation is constructed using an information set selected by the forecaster. The information set should be available at time t. The notation used for forecasts of S_{t+T} is:

 $E_t[S_{t+T}],$

where $E_t[.]$ represent an expectation taken at time t.

Each forecast has an associated *forecasting error*, ε_{t+T} . We will define the forecasting error as:

$$\varepsilon_{t+T} = S_{t+T} - \operatorname{Et}[S_{t+T}]$$

The forecasting error will be used to judge the quality of the forecasts. A typical metric used for this purpose is the Mean Square Error or MSE. For T = 1, the MSE is defined as:

MSE =
$$[(\varepsilon_{t+1})^2 + (\varepsilon_{t+2})^2 + (\varepsilon_{t+3})^2 + \dots + (\varepsilon_{t+0})^2]/Q$$
,

where Q is the number of forecasts. We will say that the higher the MSE, the less accurate the forecasting model.

There are two kinds of forecasts: *in-sample* and *out-of-sample*. The first type of forecasts works within the sample at hand, while the latter works outside the sample.

In-sample forecasting does not attempt to forecast the future path of one or several economic variables. In-sample forecasting uses today's information to forecast what today's spot rates should be. That is, we generate a forecast within the sample (*in-sample*). The fitted values estimated in a regression are in-sample forecasts. The corresponding forecast errors are called *residuals* or *in-sample forecasting errors*.

On the other hand, out-of-sample forecasting attempts to use today's information to forecast the future behavior of exchange rates. That is, we forecast the path of exchange rates outside of our sample. In general, at time t, it is very unlikely that we know the inflation rate for time t+1. That is, in order to generate out-of-sample forecasts, it will be necessary to make some assumptions about the future behavior of the fundamental variables.

Summary: Fundamental Forecasting Steps

- (1) Selection of Model (for example, PPP model) used to generate the forecasts.
- (2) Collection of St, Xt (in the case of PPP, exchange rates and CPI data needed.)
- (3) Estimation of model, if needed (regression, other methods)
- (4) Generation of forecasts based on estimated model. Assumptions about Xt+T may be needed.
- (5) Evaluation. Forecasts are evaluated. If forecasts are very bad, model must be changed.

Example V.1: In-sample Forecasting Exchange Rates with PPP

Suppose you work for a U.S. firm. You are given the following quarterly CPI series in the U.S. and in the U.K. from 2008:1 to 2009:3. The exchange rate in 2008:1 is equal to 1.9754 USD/GBP. You believe that this exchange rate, 1.5262 USD/GBP, is an equilibrium rate. Your job is to generate equilibrium exchange rates using PPP. In order to do this, you do quarterly in-sample forecasts of the USD/GBP exchange rate using relative PPP. That is,

$$E_{t}[s_{t+1}] = s_{t+1}^{F} = \frac{s_{t+1}^{F}}{s_{t}} - 1 \approx I_{d,t+1} - I_{f,t+1}$$

The forecasted level of the exchange rate USD/GBP for next period is given by $E_t[S_{t+1}]=S^{F}_{t+1=}$:

 $E_{t}[S_{t+1}] = S_{t} * [1 + \bar{S}_{t+1}] = S_{t} * [1 + (I_{US,t+1} - I_{UK,t+1})]$

Date	CPI U.S.	CPI U.K.	Inflation U.S. (I _{US})	Inflation U.K. (I _{UK})	$\begin{array}{c} \text{In-Sample} \\ \text{Forecast} \ (S^{\text{F}}_{t+1}) \end{array}$	Actual (St)	$\begin{array}{c} \text{Forecast Error} \\ \epsilon_{t+1} = S^F_{t+1} \text{-} S_{t+1} \end{array}$
2008.1	108.6	106.2	-	-		1.9754	-
2008.2	111.0	108.2	0.0221	0.019091	1.9813	1.9914	-0.0100
2008.3	112.3	109.3	0.0117	0.009813	1.9951	1.7705	0.2246
2008.4	109.1	108.4	-0.0285	-0.00795	1.7341	1.4378	0.2964
2009.1	108.6	106.1	-0.0046	-0.02137	1.4619	1.4381	0.0237
2009.2	109.7	106.9	0.0101	0.007279	1.4422	1.6481	-0.2059
2009.3	110.5	107.8	0.0073	0.009033	1.6452	1.5990	0.0463

Some calculations for $S^{F}_{2008:2}$ and $S^{F}_{2008:3}$:

1. Forecast $S_{2008:2}^{F}$. $I_{US,2008:2} = (USCPI_{2008:2}/USCPI_{2008:1}) - 1 = (111.0/108.6) - 1 = 0.0221.$ $I_{UK,2008:2} = (UKCPI_{2008:2}/UKCPI_{2008:1}) - 1 = (108.2/106.2) - 1 = 0.0191.$ $s_{2008:2}^{F} = I_{US,2008:2} - I_{UK,2008:2} = 0.0221 - 0.0191 = 0.0030.$ $S_{2008:2}^{F} = S_{2008:1}^{F} x [1 + s_{2008:2}^{F}] = 1.9754 USD/GBP x [1 + (0.0030)] = 1.9813 USD/GBP.$ $\varepsilon_{2008:2} = S_{2008:2}^{F} - S_{2008:2} = 1.9813 - 1.9914 = -0.01.$

2. Forecast $S^{F}_{2008:3}$. $S^{F}_{2008:3} = S_{2008:2} x [1 + s^{F}_{2008:3}] = 1.9914 \text{ USD/GBP} x [1 + (0.0019)] = 1.9951 \text{ USD/GBP}.$ $\epsilon_{2008:3} = S^{F}_{2008:3} - S_{2008:3} = 1.9951 - 1.7705 = 0.2246.$

3. Evaluation of forecasts. MSE: $[(-0.01)^2 + (0.2246)^2 + (0.2964)^2 + \dots + (0.0463)^2]/6 = 0.0306$

Now, you can generate trading signals. According to this PPP model, the equilibrium exchange rate in 2008:2

should be 1.9813 USD/GBP. The market price, however, is 1.9914 USD/GBP. That is, the market is valuing the GBP higher than your fundamental model. Suppose you believe that the difference (1.9813-1.9914) is due to miss-pricing factors, then you will generate a sell GBP signal. ¶

In general, practitioners will divide the sample in two parts: a longer sample (*estimation period*) and a shorter sample (*validation period*). The estimation period is used to select the model *and* to estimate its parameters. Suppose we are interested in one-step-ahead forecasts. The one-step-ahead forecasts made in this period are in-sample forecasts, not "true forecasts." These one-step-ahead forecasts are just fitted values. The corresponding forecast errors are called *residuals*.

The data in the validation period are not used during model and parameter estimation. One-stepahead forecasts made in this period are "true forecasts," often called *backtests*. These true forecasts and their error statistics are representative of errors that will be made in forecasting the future. A forecaster will use the results from this validation step to decide if the selected model can be used to generate outside the sample forecasts.

Figure V.1 shows a typical partition of the sample. Suppose that today is March 2015 and a forecaster wants to generate monthly forecasts until January 2016. The estimation period covers from February 1978 to December 2009. Different models are estimated using this sample. Based on some statistical measures, the best model is selected. The validation period covers from January 2010 to March 2015. This period is used to check the forecasting performance of the model. If the forecaster is happy with the performance of the forecasts during the validation period, then the forecaster will use the selected model to generate out-of-sample forecasts.



Figure V.1: Estimation, Validation & Out-of-sample Periods.

Example V.2: Out-of-sample Forecasting Exchange Rates with PPP Go back to Example V.1. Now, you want to generate out-of-sample forecasts. You need to make some assumptions about the future behavior of the inflation rate. *(A) Naive assumption:* $E_t[I_{t+1}] = I_{t-1}^F = I_t$.

You can generate out-of-sample forecasting by assuming that today's inflation is the best predictor for tomorrow's inflation. That is, $E_t[I_{t+1}] = I_t$.

This "naive" forecasting model leads us to a simplified version of the Relative PPP:

$$E_{t}[s_{t+1}] = s_{t+1}^{F} = (E_{t}[S_{t+1}]/S_{t}) - 1 \approx I_{d,t} - I_{f,t}$$

With the above information we can predict $S_{2008:3}$: and, then, calculate the forecast error, $\varepsilon_{2008:3}$:

$$\begin{split} s^{F}_{2008:3} &= I_{US,2008:2} - I_{UK,2008:2} = 0.0221 - 0.0191 = 0.0030. \\ s^{F}_{2008:3} &= s_{2008:2} \; x \; \begin{bmatrix} 1 + s^{F}_{2008:3} \end{bmatrix} = 1.9914 \; USD/GBP \; x \; \begin{bmatrix} 1 + (0.0030) \end{bmatrix} = 1.99735 \; USD/GBP. \\ \epsilon_{2008:3} &= s^{F}_{2008:3} - s_{2008:3} = 1.99735 - 1.7705 = 0.2269. \end{split}$$

(B) Autoregressive model: $E[I_{t+1}] = \alpha_0 + \alpha_1 I_t$.

More sophisticated out-of-sample forecasts can be achieved by estimating regression models, using survey data on expectations of inflation, etc. For example, consider the following regression model:

$$\begin{split} I_{US,t} &= \alpha^{US}_{0} + \alpha^{US}_{1} \ I_{US,t-1} + \epsilon_{US,t}. \\ I_{UK,t} &= \alpha^{UK}_{0} + \alpha^{UK}_{1} \ I_{UK,t-1} + \epsilon_{UK,t}. \end{split}$$

This autoregressive model can be estimated using historical data, say 1978:1-2008:1. Then, we have 119 quarterly inflation rates for both series. We estimate both equations.

(1) Excel output for autoregressive model for the US.

Regression Statistics						
Multiple R	0.715136					
R Square	0.51142					
Adjusted R						
Square	0.507244					
Standard Error	0.005517					
Observations	119					

ANOVA

	.15			_	Significance
	at	55	MS	F	F
Regression	1	0.003727	0.00373	122.469	6.48E-20
Residual	117	0.003561	3.04E-05		
Total	118	0.007288			

		Standard			
	Coefficients	Error	t Stat	P-value	Lower 95%
Intercept	0.00292	0.00082	3.55993	0.00054	0.001295
X Variable 1	0.700106	0.063263	11.0666	6.48E-20	0.574817

Regression	Statistics				
Multiple R	0.416771				
R Square	0.173698				
Adjusted R					
Square	0.166635				
Standard Error	0.011067				
Observations	119				
ANOVA					
					Significance
	df	SS	MS	F	F
Regression	1	0.003013	0.00301	24.5947	2.42E-06
Residual	117	0.014331	0.00012		
Total	118	0.017344			
		Standard			
	Coefficients	Error	t Stat	P-value	Lower 95%
Intercept	0.007128	0.001455	4.89963	3.12E-06	0.004247
X Variable 1	0 44 4970	0 002555	1 05020	2 425 06	0 240000

(2) Excel output for autoregressive model for the UK.

That is, we obtain the following estimated coefficients: α^{US}_0 =.00292, α^{US}_1 =.7001, α^{UK}_0 =.00713, and α^{UK}_1 =.4144.

First, you evaluate the regression by looking at the t-statistics and the R^2 . The t-statistic is used to test the null hypothesis that a coefficient is equal to zero. The R^2 measures how much of the variability of the dependent variables is explained by the variability of the independent variables. That is, the R^2 measures the explanatory power of our regression model. Both R^2 coefficients are far from zero, relatively high for the U.S. inflation rate (51%). All coefficients have a t-stats higher than 1.96. That is, you will say that they are significant at the 5% level –i.e., with p-values smaller than .05. That is, all the coefficients are statistically different from zero.

Second, you use the regression to forecast inflation rates. Then, you will use these inflation rate forecasts to forecast the exchange rate. That is,

$$\begin{split} I^{F}_{US,2008:3} &= .00292 + .7001 \ x \ (.0221) = .01839 \\ I^{F}_{UK,2008:3} &= .00713 + .4144 \ x \ (.0191) = .01505 \\ s^{F}_{2008:3} &= I^{F}_{US,2008:3} - I^{F}_{UK,2008:3} = .01839 - 01505 = .00334. \\ S^{F}_{2008:3} &= 1.9914 \ USD/GBP \ x \ [1 + (.00334)] = 1.99802 \ USD/GBP. \\ \epsilon_{2008:3} &= S^{F}_{2008:3} - S_{2008:3} = 1.99802 - 1.7705 = 0.22752. \end{split}$$

That is, you predict, over the next quarter, an appreciation of the GBP. You can use this information to manage currency risk at your firm. For example, if, during the next quarter, the U.S. firm you work for expects to have GBP outflows, you can advise management to hedge. ¶

Example V.3: Out-of-sample Forecasting Exchange Rates with a Structural Ad-hoc Model Suppose a Malaysian firm is interested in forecasting the MYR/USD exchange rate. This Malaysian firm is an importer of U.S. goods. A consultant believes that monthly changes in the MYR/USD exchange rate are driven by the following econometric model (MYR = Malaysian Ringitt):

$$s_{MYR/USD,t} = \alpha_0 + \alpha_1 \left(I_{MYR} - I_{USD} \right)_t + \alpha_2 \left(y_{MY} - y_{US} \right)_t + \varepsilon_t \tag{V.1}$$

where, $(I_{MYR} - I_{USD})_t$ represents the inflation rate differential between Malaysia and the U.S., and $(y_{MY} - y_{US})_t$ represents the income growth rates differential between Malaysia and the U.S. The spot rate this month is S_t=3.1021 MYR/USD. Suppose equation (V.1) is estimated using 10 years of monthly data with ordinary least squares (OLS). We have the following excel output:

SUMMARY OUTPUT	tistics				
Multiple R	0.092703				
R Square	0.018594				
Adjusted R Square	-0.0087				
Standard Error	0.051729				
Observations	112				
ANOVA					
	df	SS	MS	F	Significance F
Regression	2	0.002528	0.00126	0.47242	0.624762
Residual	109	0.291666	0.00268		
Residual Total	109 111	0.291666 0.294195	0.00268		
Residual Total	109 111	0.291666 0.294195	0.00268		
Residual Total	109 111	0.291666 0.294195 Standard	0.00268		
Residual Total	109 111 <i>Coefficients</i>	0.291666 0.294195 Standard Error	0.00268 t Stat	P-value	
Residual Total Intercept	109 111 <i>Coefficients</i> 0.006934	0.291666 0.294195 Standard Error 0.005175	0.00268 t Stat 1.3399	<i>P-value</i> 0.18028	
Residual Total Intercept (I _{MYR} – I _{USD}) _t	109 111 <i>Coefficients</i> 0.006934 0.215927	0.291666 0.294195 Standard Error 0.005175 0.105824	0.00268 <i>t Stat</i> 1.3399 2.04044	<i>P-value</i> 0.18028 0.04131	

That is, the coefficient estimates are: $a_0 = 0.00693$, $a_1 = 0.21593$, and $a_2 = 0.09159$. That is, the output from your OLS regression is:

 $E[s_{MYR/USD,t}] = \begin{array}{c} 0.00693 + 0.21593 (I_{MYR} - I_{USD})_t + \\ (1.34) (2.04) (1.77) \end{array} + \begin{array}{c} 0.09159 (y_{MY} - y_{US})_t, \\ (1.77) \end{array}$

Let's evaluate our ad-hoc model. The t-statistics (in parenthesis) for the two variables are all bigger than 1.65. Therefore, all the explanatory variables are statistically significant at the 10% level. This regression has an R² equal to .0186. That is, $(I_{MYR} - I_{USD})_t$ and $(y_{MY} - y_{US})_t$ explain less than two percent of the variability of changes in the MYR/USD exchange rate. This is not very high, but the t-stats give some hope for the model. The t-statistics (in parenthesis) for the two variables are all bigger than 1.65. Therefore, all the explanatory variables are statistically significant at the 10% level. The Malaysian firm decides to use this model to generate out-of-sample forecasts.

Suppose the Malaysian firm has forecasts for next month for $(I_{MYR} - I_{USD})_t$ and $(y_{MY} - y_{US})_t$: 3% and 2%, respectively. Then,

 $s^{F}_{MYR/USD,t+one-month} = 0.00693 + 0.21593 * (0.03) + 0.09159 * (0.02) = .0152.$

The MYR is predicted to depreciate 1.52% against the USD next month. The spot rate this month is S_t=3.1021 MYR/USD, then, for next month, we predict:

 $S_{t+1}^F = 3.1021$ MYR/USD (1 + .0152) = 3.1493 MYR/USD.

Based on these results, the Malaysian firm, which imports goods from the U.S., decides to hedge its next month USD anticipated outflows.

♦ Some Practical Issues in Fundamental Forecasting

There are several practical issues associated with any fundamental analysis forecasting, such as the forecasting model of equation (V.1):

(1) Correct specification. That is, are we using the "right model?" (In econometrics jargon, "correct specification.")

(2) Estimation of the model. This is not a trivial issue. For example, in equation (V.1) we need to estimate the model to get a_0 , a_1 , and a_2 . Bad estimates of a_0 , a_1 , and a_2 will produce a bad forecast for $s_{MYR/USD,t+one-month}$. This issue sometimes is related to (1).

(3) Contemporaneous variables. In a model like equation (V.1), some of the explanatory variables are contemporaneous. We also need a model to forecast the contemporaneous variables. For example, in the equation (V.1) we need a model to forecast INT_t and INC_t. In econometrics jargon, this is called simultaneous equations models. \blacklozenge

1.A.2 Fundamental Approach: Evidence

In 1979 Richard Levich, from New York University, made a comparison of the forecasting performance of several models for the magazine *Euromoney*. Levich studied the forecasting performance of various services relative to the forward rate. The forward rate is a free, observable forecast. A good forecaster has to beat the forward rate.

Let $S^{F}_{j,i}$ be the forecasting service j's forecasted exchange rate for time t=i and S_i the realized exchange rate at time t=i. Let $F_{t,i}$ denote the forward rate for time T=i.

Levich compared the mean absolute error (MAE) of the forecasting service and the forward market, defined as

$$MAE_{FSj} = \{ |S_{t+1}^F - S_{t+1}| + |S_{t+2}^F - S_{t+2}| + ... + S_{t+n}^F - S_{t+n}| \} * (1/n),$$

$$MAE_{FM} = \{ |F_{t,1} - S_{t+1}| + ||F_{t+1,2} - S_{t+2}| + ... + ||F_{t+n-1,n} - S_{t+n}| \} * (1/n).$$

For the sample period 1977-1980 and for twelve major forecasting services, Levich compared the percentage of times the services did better than the forward market, that is $MAE_{FS} < MAE_{FM}$. The results were:

Time Horizon	% of Observations with MAE _{FS} < MAE _{FM}
1 month	28.8%
3 months	24.0%
6 months	28.8%
12 months	32.7%

In terms of mean absolute error, on average the forecasting services underperformed the forward rate more than two-thirds of the time. Since observation of the forward rate is free and since, as mentioned above, the forward rate is not an unbiased forecast of the future spot rate, these results suggest that paying a forecasting service might not be a wise expense. However, Levich found that forecasting services have some ability to predict whether the future spot rate will be greater than or smaller than the current forward rate. For some investors the *direction* of the forecast maybe as important as the absolute or squared error.

Example V.4: Suppose the current one-month forward rate is $F_{t,1-mo} = .7335$ USD/CAD and the forecast for the future spot rate is .7342 USD/CAD. Based on the forecasted CAD appreciation over the forward rate, Ms. Sternin, a U.S. speculator, decides to buy CAD forward. Suppose that the CAD appreciates to $S_{t+1-mo} = .7390$ USD/CAD. The direction of the forecast (relative to the forward rate) was right, the CAD appreciated over the forward rate.

Then the (absolute) size of the forecast error in this case is: .7390 - .7342 = .0052 USD/CAD.Ms. Sternin would not be unhappy with the forecasting error, since she made a profit of: .7390 - .7335 = .055 USD/CAD.

On the other hand, suppose that the CAD depreciates to $S_{t+1-mo} = .7315$ USD/CAD. Now, the direction of the forecast (relative to the forward rate) was wrong.

The (absolute) size of the error was smaller: |.7315 - .7342| = .0027 USD/CAD.But, Ms. Sternin would have lost: .7315 - .7335 = -.0020 USD/CAD.

Therefore, Ms. Sternin might prefer to have a forecast that is correct in direction, and might not care about the MAE. \P

In 1980, Levich tested the number of times a profit is made from taking advantage of the direction forecasted by some forecasting services. He found that some forecasting services were able to

correctly predict the direction more than 50 percent of the time. In 1982 and 1983, Levich repeated the same study and found that the forecasting services that performed well in 1980 were not the same as those that did well in the updated study. Therefore, it seems that it is very difficult to consistently predict the future spot rate relative to the forward rate.

♦ Forecasting: "Que sera, sera... the future's not ours to see"

Forecasting exchange rates is very difficult. As mentioned above, forward rates are poor predictors of future spot rates. Forecasting services on average have a difficult time just beating the forward rate. The random walk tends to outperform the other foreign exchange rate models. Paraphrasing a popular American TV show from the fifties and early sixties, we can say: "Doris Day knows best." \blacklozenge

1.B <u>Technical Approach</u>

The *technical approach* (TA) focuses on a smaller subset of the available data. In general, it is based on price information. The analysis is "technical" in the sense that it does not rely on a fundamental analysis of the underlying economic determinants of exchange rates or asset prices, but only on extrapolations of past price trends. Technical analysis looks for the repetition of specific price patterns. Technical analysis is an art, not a science.

Computer models attempt to detect both major trends and critical, or turning, points. These turning points are used to generate trading signals: buy or sell signals.

The most popular TA models are simple and rely on moving averages (MA), filters, or momentum indicators.

1.B.1 <u>Technical Analysis Models</u>

1.B.1.i <u>MA Models</u>

The goal of a MA model is to smooth erratic daily swings of asset prices in order to signal major trends. A MA is simply an average of past prices. We will use the simple moving average (SMA). An SMA, S_t^{MA} , is the unweighted mean of the previous Q data points:

$$S_t^{MA} = (S_t + S_{t-1} + S_{t-2} + \dots + S_{t-(Q-1)})/Q$$

If we include the most recent past prices, then we calculate a short-run MA (SRMA). If we include a longer series of past prices, then we calculate a long-term MA (LRMA). The *double MA system* uses two S_t^{MA} 's: a LRMA and a SRMA. A LRMA will always lag a SRMA because it gives a smaller weight to recent movements of exchange rates.

In MA models, buy and sell signals are usually triggered when a SRMA of past rates crosses a LRMA. For example, if a currency is moving downward, its SRMA will be below its LRMA. When it starts rising again, it soon crosses its LRMA, generating a buy foreign currency signal.



Buy FC signal: When SRMA crosses LRMA from below. Sell FC signal: When SRMA crosses LRMA from above.

Example V.5: Generating trading signals for the (USD/GBP) using the Double MA model. We generate a SRMA using 30 days of information (red line) We generate a LRMA using 150 days of information (green line).

Every time there is a crossing, the double MA model generates a trading signal.



The double MA model generates many trading signals, as indicated by the crossings between the SRMA (red line) and the LRMA (green line). For example, there is a sell GBP signal in late 2007. By April 2009, the model generates a buy GBP signal. ¶

1.B.1.ii Filter Models

This is probably the most popular TA model. It is based on the finding that asset prices show significant small autocorrelations. If price increases tend to be followed by increases and price decreases tend to be followed by decreases, trading signals can be used to profit from this autocorrelation. The key of the system relies on determining when exchange rates start to show significant changes, as opposed to irrelevant noisy changes. Filter methods generate buy signals when an exchange rate rises X percent (the filter) above its most recent trough, and sell signals when it falls X percent below the previous peak. Again, the idea is to smooth (filter) daily fluctuations in order to detect lasting trends. The filter size, X, is typically between 0.5% and 2.0%.

Example V.6: Determination of Trading signals with a filter model. Let the filter, X, be $1\% \Rightarrow X=1\%$.

First, we need to determine a peak or a through. Then, we generate trading signals.





Trough = 1.349 CHF/USD (X = CHF .01349) \Rightarrow When St crosses 1.36249 CHF/USD, Buy USD.

Note that there is a trade-off between the size of the filter and transaction costs. Low filter values, say 0.5%, generate more trades than a large filter, say 2%. Thus, low filters are more expensive than large filters. Large filters, however, can miss the beginning of trends and then be less profitable.

1.B.1.iii <u>Momentum Models</u>

Momentum models determine the strength of an asset by examining the change in velocity of the movements of asset prices. If an asset price climbs at significant increasing speed, a buy signal is issued. The trader needs to determine what constitutes "significant" increasing speed.

1.B.1.iv <u>Newer Models</u>

These models monitor the derivative (slope) of a time series graph. Signals are generated when the slope varies significantly. There is a great deal of discretionary judgement in these models. Signals are sensitive to alterations in the filters used, the period length used to compute MA models and the method used to compute rates of change in momentum. These facts can make two TA practitioners using the same model, but different parameters, to generate different signals.

To solve this problem, there are several newer TA methods that use more complicated mathematical formulas to determine when to buy/sell, without the subjectivity of selecting a parameter. Clements (2010, *Technical Analysis in FX Markets*) describes four of these methods: Relative strength indicator (RSI), Exponentially weighted moving average (EWMA), Moving average convergence divergence (MACD) and (iv) Rate of change (ROC).

1.B.2 <u>Technical Approach: Evidence</u>

In Chapter III we presented evidence that for out-of-sample forecasts, the random walk model seems to be as good as any other model. Therefore, it should not be a surprise that many economists have a strong negative view of technical analysis forecasts. The usual argument against technical analysis is based on the concept of market efficiency (discussed in the Review Chapter). The efficiency argument says that there cannot be reliable price patterns in exchange rates. If there were reliable price patterns, traders would use them, and, then, by using them, traders would destroy them. The marketplace, however, is full of newsletters and consultants selling technical analysis forecasts and predictions.

Recall that the random walk model makes the exchange rate independent of its past history. Empirical studies that test the efficiency of exchange rate markets usually employ simple linear regression analysis. John F. O. Bilson (1990) points out that linear comparisons are meaningless, since technical analysts rely on nonlinearities.

For example, Bilson estimates the following model:

$$y = b_1 X_1 + b_2 X_2 + b_3 X_3 + u$$
,

where

 $\begin{array}{ll} y = (S_{t+1} - F_{t,1})/F_{t,1}, & \mbox{realized ex-post returns on the currency}, \\ X_1 = (F_{t,1} - S_t)/F_{t,1}, & \mbox{a yield indicator (interest rate differential)}, \\ X_2 = (S_t - F_{t-3,1})/F_{t-3,1}, & \mbox{a trend indicator}, \\ X_3 = X_2 \exp(|X_2|/\text{std}(X_2)), & \end{array}$

where $F_{t,1}$ is the one-month exchange rate forecast. The third variable accounts for the nonlinear pattern of serial correlation by discounting large trend values.

Bilson collected New York interbank spot and one-month forward rates, from April 1975 to April 1991, on the three major currencies -GBP, DEM, and JPY. The estimates of the above equation are (t-statistics in parentheses):

$\hat{\mathbf{Y}} =$	-1.0041 X ₁	-	.0939 X2	+	.6009 X ₃ .
	(3.03)		(2.66)		(5.10)

All the coefficients are significantly different from zero at the 1, which shows that past information has some predictive power.

Bilson later examined the forecast ability of the above model. Let \hat{Y} denote the fitted value from the regression estimated above. He regressed y against \hat{Y} , that is,

 $y_t = a + b \hat{Y}_t + \varepsilon_t.$

The regression results, for the three major currencies, are given below:

Currency	а	b	R ²
GBP	1071 (0.41)	1.0948 (3.26)	.052,
DEM	.2436 (0.41)	1.2637 (3.64)	.065,
JPY	.5911 (0.41)	1.4089 (4.23)	.085.

The expected returns estimated by Bilson account for between 5% and 8% of the monthly variation in the ex-post realized returns on the currency. It should be noted, however, that the R^{2} 's are small. Therefore, one should be very careful in the interpretation of the above results.

Bilson argues that these results demonstrate that the expected returns are a statistically significant indicator of actual returns. The relation between past trends and the present might be non-linear. His model predictions are entirely based on past price information. He argues in favor of technical analysis and non-linear filters.

Many practitioners claim that technical analysis has value since it can be useful to forecast exchange rates, especially in the short term. They claim that, using technical analysis forecasts, profitable trading strategies can be constructed. In general, in-sample results tend to be good –i.e., profitable– for TA strategies in the FX markets. For example, an old paper by Richard Sweeney (1986), published in the Journal of Finance, investigates the power of technical forecasting. He finds that simple filter rules were able to generate excess returns during the period 1973-1980. For example, Sweeney estimated that a 1% filter rule generated a return of 2.8% while a buy-and-hold strategy generated a 1.6% return. But, the out of sample performance of TA models is weak. LeBaron (1999) speculates that the apparent success of TA in the FX market is influenced by the periods where there is CB intervention. There is also some evidence that, even in-sample, the performance of TA models has declined over time. Ohlson (2004) finds that the profitability of TA strategies in the FX market have significantly declined over time, with about zero profits by the 1990s.

In a 2007 survey, Park and Irwin (2007) published in the Journal of Economic Surveys, review the TA

recent literature in different markets. They report that out of 92 modern academic papers, 58 found that TA strategies are profitable. Park and Irwin point out, however, many problems with most studies: data snooping, ex-post selection of trading rules, difficulties in the estimation of risk and transaction costs.

II. Looking Ahead: Exchange Rate Volatility

We have seen that exchange rates are very difficult to forecast. This difficulty makes it difficult to estimate the future value assets and liabilities denominated in foreign currency. In finance, uncertainty about the value of securities creates risk. Many financial models associate the variance of an asset with an asset's riskiness. Investors require higher rates of return for riskier assets. The volatility of an asset is also important in other fields of finance. For example, option pricing formulas use as a vital input an estimate for the variance.

For many years, researchers assumed that the variance of asset returns was constant. The assumption of a constant variance is called homoscedasticity. This assumption is a common assumption in econometrics. For example, in the derivations of Appendix III we state that the variance of the error term is constant. Homoscedasticity is a convenient assumption. It simplifies the estimation of time series models.

Variances of asset returns, however, tend to be time-varying. That is, the returns of financial assets are heteroscedastic over time -i.e., non-constant variance. For example, Figure V.2 plots on the first panel the weekly CHF/USD exchange rate and on the second panel the CHF/USD percent changes. There are periods where exchange rates are quite volatile, while during other periods exchange rates do not move very much. For example, consider the volatile 1978-1980 period with the more quiet 2006-2007 period. It is easily observed that the assumption of a constant variance does not hold for the CHF/USD exchange rate.

FIGURE V.2 CHF/USD Levels and Changes (January 1971-January 2010)



Many researchers found the variance of asset returns to be predictable over time. The problem faced by researchers was to find an appropriate model that incorporated the empirical characteristic of financial assets returns. Robert F. Engle, in a paper published in <u>Econometrica</u> in 1982, introduced the Autoregressive Conditional Heteroskedastic (ARCH) model. This model became very popular because of its simplicity and intuitive appeal. Engle's ARCH(q) model is specified as:

$$\sigma^2_t = \alpha_0 + \sum_{i}^q \alpha_i \ e^2_{t-i}.$$

The variance at time t depends on past conditional errors. Therefore, the variance at time t is predictable with the information set available at time t-1. This information set will be labeled Ψ_{t-1} . The ARCH(q) model has a familiar time-series structure. If we consider the squared error, e^2_t , as an approximation for the variance at time t, the ARCH(q) model has a similar structure than the AR(q) model (see Appendix V).

(V.2)

Empirical Regularities and ARCH Models

Rob Engle's model incorporates an empirical fact observed in returns of financial assets: large (small) changes in returns are likely to be followed by large (small) changes in returns. A large shock yesterday (e_{t-1} is big) tends to increase the variance today, σ^2_t . This phenomenon is called volatility clustering and it is typical of financial assets. Volatility clustering is easily observed in the second panel of Figure V.2. Another empirical fact that the ARCH(q) model incorporates is that the unconditional distribution of financial assets' returns has thicker (fatter) tails than the normal distribution. That is, in statistical language, the unconditional distribution is *leptokurtic*.

Later, Tim Bollerslev, in a paper published in the <u>Journal of Econometrics</u>, in 1986, introduced the GARCH model, which generalized the ARCH model. The GARCH(q,p) model is specified as follows:

$$\sigma^{2}_{t} = \alpha_{0} + \Sigma_{i}^{q} \alpha_{i} e^{2}_{t-i} + \Sigma_{i}^{p} \beta_{i} \sigma^{2}_{t-i}.$$
(V.3)

Now, in equation (V.3), the variance at time t depends on past errors and past variances. It can be shown that the GARCH(q,p) is similar to an ARMA model.

Since a variance has to be positive, the restrictions $\alpha_0 \ge 0$, $\alpha_i \ge 0$ (i=1,...,q), and $\beta_i \ge 0$ (i=1,...,p) are usually imposed. If we take expectations in equation (V.3), and set $E[e^2_t]=E[\sigma^2_t]=\sigma^2$, we can obtain the unconditional (or average) variance, σ^2 , that is,

$$\sigma^2 = \alpha_0 / (1 - \Sigma_i^q \alpha_i - \Sigma_i^p \beta_i).$$

For the unconditional variance to be well defined (i.e., bounded and positive), we need the following condition:

 $\sum_{i}^{q} \alpha_{i} + \sum_{i}^{p} \beta_{i} < 1.$

GARCH models have been successfully employed for exchange rates and other financial series at different frequencies: monthly, weekly, daily and intradaily. In several of these applications a useful regularity has emerged: in general, a GARCH(1,1) specification is able to capture the dynamics of the conditional variance:

$$\sigma^{2}_{t} = \alpha_{0} + \alpha_{1} e^{2}_{t-1} + \beta_{1} \sigma^{2}_{t-1}.$$
(V.4)

The GARCH(1,1) model says that the current variance estimate can be calculated from the variance estimated last period, σ^2_{t-1} , modified by the squared error also observed last period. In the GARCH(1,1) the unconditional variance is well defined when $\alpha_1+\beta_1<1$. Let $\lambda=\alpha_1+\beta_1$. For reasons that will become clear below, the parameter λ is called the persistence parameter.

Estimation of GARCH models is usually done using a method called maximum likelihood estimation. To use this method of estimation, we have to assume a distribution for the conditional errors, e_t . We will assume that e_t , conditional on the information set Ψ_{t-1} , follows a normal distribution with mean

zero and variance σ^2_t . That is, $e_t | \Psi_{t-1} \sim N(0, \sigma^2_t)$.

Given the GARCH structure, the estimation involves nonlinear optimization techniques. Sometimes, it is difficult to obtain convergence for the GARCH model. For those familiar with nonlinear optimization, it is well-known that initial values are very important. Many statistical packages, like SAS and TSP, estimate GARCH models. Once the parameters α_i 's and β_i 's are estimated, equation (V.3) can be used to forecast the next-period variance.

Example V.5: Sietes Swiss, a Swiss company, wants estimate the volatility of the weekly CHF/USD exchange rate. From their experience, forecasters at Sietes Swiss know that changes in exchange rates are very difficult to forecast. Therefore, they use a very simple AR(1) process for the mean return. The variance is modeled using the popular GARCH(1,1) specification:

$$\begin{split} s_t &= [log(S_t) - log(S_{t-1})] x 100 = a_0 + a_1 s_{t-1} + e_{Ft}, \qquad e_t | \Psi_{t-1} \sim N(0, \sigma^2_t). \\ \sigma^2_t &= \alpha_0 + \alpha_1 \; e^2_{t-1} + \beta_1 \; \sigma^2_{t-1}. \end{split}$$

Sietes Swiss using SAS estimates the above model using weekly data provided by the Federal Reserve of Chicago (http://www.frbchi.org). The sample starts in January 8, 1971 and ends in January 21, 2010, for a total of 2038 observations.

The estimated model for st is given by:

$$\begin{split} s_t &= \ \text{-}0.055 + \ 0.051 \ s_{t\text{-}1}, \\ (0.32) & (1.66) \\ \sigma^2_{\ t} &= \ 0.236 + \ 0.092 \ e^2_{\ t\text{-}1} + \ 0.822 \ \sigma^2_{\ t\text{-}1}, \\ (1.71) & (4.00) & (11.83) \end{split}$$

where t-statistics are in parenthesis.

In Figure V.3 we plot the conditional volatility (squared root of the variance) over time for the CHF/USD exchange rate, as calculated by the estimated GARCH(1,1) model. Consistent with the visual intuition in Figure V.1, volatility changes over time. We observe the high volatility periods associated with the 1978-1980



FIGURE V.3 GARCH(1,1) CHF/USD Volatility Estimates (1/1971-1/2010)

Note that Sietes Swiss finds that $\lambda = .955 < 1$. That is, the unconditional variance, σ^2 , is well defined. Given that $e_{1048} = -0.551$, and $\sigma^2_{2038} = 2.554$, Sietes Swiss is able to forecast the variance for the week ending on January 28, 2010 $-\sigma^2_{2039}$. That is,

 $\sigma^{2}_{1039} = 0.140 + 0.079 (-0.551)^{2} + 0.876 (2.554) = 2.401.$

Multi-period variance forecasts can be easily calculated using a GARCH(1,1) model. It can be shown that the conditional expectation of the variance k periods ahead is given by

 $E_t[\sigma^2_{t+k}] = (\alpha_1 + \beta_1)^k \left[\sigma^2_t - \alpha_0/(1 - \alpha_1 - \beta_1)\right] + \alpha_0/(1 - \alpha_1 - \beta_1) = \lambda^k \left[\sigma^2_t - \sigma^2\right] + \sigma^2$

Note that if $\lambda = (\alpha_1 + \beta_1) < 1$, as $k \to \infty$, $\lambda^k \to 0$. That is, as we forecast more into the future, i.e., $k \to \infty$, the forecasts converges to the unconditional variance, σ^2 . Note that the parameter λ represents the rate of convergence (decay) of the conditional variance towards σ^2 .

Example V.6: Suppose Cannigia Co. estimates a GARCH(1,1) model using 300 daily observations for the GBP/USD exchange rate. Cannigia Co. uses percentage changes of the USD/GBP exchange rates to estimate the GARCH model. Cannigia Co. obtains the following estimates for the variance parameters : $\alpha_0 = .175$, $\alpha_1 = .123$, and $\beta_1 = .847$. The estimate for the conditional variance of the last observation is $\sigma^2_{300} = 1.959$. Cannigia wants to forecast the variance 10 days from now.

The unconditional variance is given by

 $\sigma^2 = .175/(1 - .123 - .847) = 5.833.$

Then, the 10-day-ahead variance forecast is:

 $E_{300}[\sigma^2_{310}] = (.97)^{10} [1.959 - 5.833] + 5.833 = 2.976.$

That is, the volatility forecast 10 days ahead is $\sigma_{310} = 1.725\%$.

Many empirical studies also find that $\lambda = \alpha_1 + \beta_1$ is close to one. For instance, in Example V.2, we find that $\lambda = .955$. When $\lambda = 1$, the GARCH(1,1) model is called integrated GARCH model, or IGARCH. An IGARCH model does not have a well-defined σ^2 . This formulation reduces the number of parameters of the conditional variance equation (V.4) to two. Therefore, (V.4) becomes

 $\sigma^2_t = \alpha_0 + \alpha_1 e^2_{t-1} + (1-\alpha_1) \sigma^2_{t-1}$.

When a series is described by an IGARCH process, the multi-period variance forecast becomes:

 $E_t[\sigma^2_{t+k}] = \sigma^2_t + k \alpha_0.$

The variance at time t affects the conditional variance forecasts into the indefinite future. Thus, today's shocks to the conditional variance persists forever in conditional variance forecasts. When λ is close to one, shocks to the variance today show a high persistence in the conditional variance forecasts. That is, a big surprise today (big et-1) has an influence on the variance for several periods.

2.A J.P. Morgan's RiskMetrics Approach

In October 1994, J.P. Morgan unveiled and made freely available its risk management software RiskMetrics (http://www.jpmorgan.com/RiskManagemet/RiskMetrics). RiskMetrics is a method that estimates the market risk of a portfolio, using the Value-at-Risk (VAR) approach. RiskMetrics uses a simple specification to model a time-varying variance:

 $\sigma^{2}_{t} = (1 - \gamma) e^{2}_{t-1} + \gamma \sigma^{2}_{t-1}.$

That is, next period's variance is a weighted-average of this period's squared forecast error and this period's variance. The weights are given by the parameter γ , which is referred to as the decay factor and, then, should be less than one.

You should note the similarity of the RiskMetrics approach with the IGARCH model, with $\alpha_0=0$. In this case, $\gamma = (1-\alpha_1) = \beta_1$. Therefore, shocks to exchange rates persist forever in variance forecasting. Based on convenience and experience, J.P. Morgan selected to set γ equal to .94 for daily data. Ideally, this parameter should be estimated for every series, every day. Given the large number of series that J.P. Morgan uses (more than 500), RiskMetrics uses the same γ for all the series. For monthly volatility, RiskMetrics sets $\gamma=.97$.

Example V.7: Suppose Cannigia Co. estimates a time-varying variance using the RiskMetrics model using 300 daily observations for the GBP/USD exchange rate. Cannigia Co. uses percentage changes of the USD/GBP exchange rates to estimate volatility. The estimate for the conditional variance of the last observation is $\sigma^2_{300} = 1.959$ and $e_{300} = 0.663$. Cannigia wants to forecast the variance tomorrow. Using RiskMetriks, Cannigia forecasts:

 $E_{300}[\sigma^2_{301}] = (.94) \ 1.959 + .06 \ (.633)^2 = 1.8655$

That is, the volatility forecast for tomorrow is $\sigma_{310} = 1.366\%$.

2.B Using Forecasts of the Variance: Value-at-Risk (VaR)

Value-at-Risk (VaR) provides a number, which measures the market risk exposure of a portfolio of a firm over a given length of time. VaR measures the maximum expected loss in a given time interval, within a confidence interval. Note that to calculate a VaR of a portfolio, we need to specify a time interval and the significance level for the confidence interval. Suppose we specify a day as the time interval and 5% as the significance level to calculate the VaR of a FX portfolio. Suppose the VaR of the FX portfolio equals to USD 10,000. This VaR amount (USD 10,000) represents the potential loss of the FX portfolio in about one every twenty days.

Example V.8: Microsoft uses a VaR computation, within a 97.5% confidence interval, to estimate the maximum potential 20-day loss in the fair value of its foreign currency denominated investments and account receivables, interest-sensitive investments and equity securities. At the end of June 2001, Microsoft calculated a VaR of negligible for foreign currency instruments, USD 363 million for interest sensitive

2.B.1 Calculating a VaR

Suppose that a firm has a portfolio of foreign currencies, with a value W. Let the returns, r, of this portfolio follow a probability distribution, with mean return of μ and volatility equal to σ . The initial value of the portfolio is W₀. The value of the portfolio at the end of the period is W₁= W₀(1 + r). Now, let W* be the minimum value of the portfolio within a confidence interval c. That is, W*= W₀(1 + r*), where r* represents the minimum return of the portfolio within a confidence interval.

Now, we can define a measure for the mean exposure:

$$VaR (mean) = E(W_1) - W^* = W_0(1+\mu) - W_0(1+r^*) = W_0 (\mu - r^*)$$

Note that computing a VaR is equivalent to computing r*.

We can derive the VaR using the distribution of the future values of the portfolio, f(w). Within a confidence interval c, we can find W* such that

$$c = \int_{W^*}^{\infty} f(w) d(w)$$

A usual assumption is to convert f(w) into a standard Normal distribution. Let's calculate the VaR (mean) using a standard Normal distribution.

Recall that a standard normal distribution has $\mu=0$ and $\sigma=1$. First, we need to standardize the data. That is, $Z = (r-\mu)/\sigma$.

Our goal is to find W*. That is, we want to find r*, which is equivalent to find $-\alpha = (r^*-\mu)/\sigma$. Note that $\alpha > 0$.

$$1 - c = \int_{-\infty^*}^{W^*} f(w)d(w) = \int_{-\infty}^{-\alpha} \Phi(\varepsilon)d(\varepsilon) = N(-\alpha)$$

For a one-tail 95% confidence interval, $\alpha = 1.65$. For a one-tail 97.5% confidence interval, $\alpha = 1.96$.

Now, we can easily find r*, for a given confidence interval: $r^* = -\alpha \sigma + \mu$

Therefore, the VaR (mean) is equal to:

VaR (mean) = $W_0(\mu - r^*) = W_0(\alpha \sigma)$

In general, μ and σ are annualized. We have to adjust these numbers to the VaR's time interval. This can be done by multiplying the numbers by Δt , which represent the VaR's frequency in years. That is, VaR can be calculated by

VaR (mean) = W_0 \alpha \sigma \sqrt{\Delta t}.

You should notice now the usefulness of a good volatility forecast. The only variables needed to calculate VaRs are variances. All the other parameters are given to us.

Example V.9: Suppose we want to calculate the VAR of a portfolio of foreign currencies. The value of the portfolio is USD 220 million. Using a GARCH model, we estimated the annualized volatility of changes in exchange rates as 12%. The time interval is ten days ($\Delta t=10/252=.03968$). We want to use a 95% confidence interval.

VaR (mean) = $W_0 \alpha \sigma \sqrt{\Delta t}$. = USD 220Mx1.65x0.12x $\sqrt{.03968}$ = USD 8,677,082.

That is, the maximum one-day loss of this portfolio is USD 8,677,082.

It is possible to calculate a VaR without calculating variances. One way to do this is to use the empirical distribution. That is, we can find W*, such that, $p = P(w \le W^*) = 1$ -c. Thus, we can calculate W* by looking at the histogram (empirical distribution). We can locate W* from the past history. For example, if we use a 95% confidence interval, W* is the value in the histogram that leaves 5% of the observations to the left.

Example V.10: Suppose we have managed the same portfolio for two years and we have recorded the value of the portfolio at the end of each day. That is, we have roughly 500 observations. If we use a 95% confidence interval, W* is calculated by looking at the 25th (500x.05) ordered observation, which is equal to USD -6.1M. The mean of the daily value of the portfolio is USD 3.2M. Thus, the daily VAR is:

 $VaR (mean) = E(W_1) - W^* = USD 3.2M - (-USD 6.1M) = USD 9.3M.$

Interesting readings:

Parts of Chapter V were based on the following books:

International Financial Markets, by J. Orlin Grabbe, published by McGraw-Hill.

International Investments, by Bruno Solnik, published by Addison Wesley.

APPENDIX V-A

BASIC FORECASTING MODELS

A.I Forecasting from Econometric Models

The econometric approach to forecasting consists first of formulating an econometric model that relates a dependent variable to a number of independent variables that are expected to affect it. The model is then estimated and used to obtain conditional or unconditional forecasts of the dependent variable. The models are generally formulated using economic theory and the statistical properties of the variables included in the model.

Example A.V.1: In Example V.3, a company believes that monthly changes in the MYR/USD exchange rate are related to the interest rates differential between Malaysia and the U.S. (INT_t) and income growth rates differential (INC_t) between Malaysia and the U.S. That is, the econometric model is given by:

 $s_{MYR/USD,t,one-month} = a_0 + \delta INT_t + \mu INC_t + \epsilon_t,$ (A.1)

where ε_t is a prediction error assumed to follow a normal distribution with zero mean and constant variance, σ^2 .

The IFE predicts that INT_t should have a positive coefficient. That is, if Malayan interst rates increase relative to U.S. interest rates, then the MYR should depreciate with respect to the USD (i.e., δ should be positive). Similarly, the Asset Approach predicts that INC_t should have a negative coefficient. That is, if income grows in Malaysia at a faster rate than in the U.S., the MYR should appreciate with respect to the USD (i.e., μ should be negative).

Several economic series seem to show seasonal effects. For example, many researchers have found a Monday effect in the U.S. stock market. Since these seasonal effects are predictable, many forecasters include seasonal variables in an econometric model like equation (A.1).

Example A.V.2: In Example A.V.1 a forecaster might like to introduce monthly seasonal variables to predict the monthly change in the MYR/USD. In this case, equation (A.1) would include eleven monthly dummy variables.

 $s_{MYR/USD,t,one-month} = a_0 + \delta \ INT_t + \mu \ INC_t + \tau_{Jan} \ D_{Jan} + ... + \ \tau_{Nov} \ D_{Nov} + \epsilon_t,$

where

A.II Forecasting from Time Series Models

Econometric models are generally based on some underlying economic model. A popular alternative to econometric models, especially for short-run forecasting is known as time series models. These models typically relate a dependent variable to its past and to random errors that may be serially correlated.

Time series models are generally not based on any underlying economic behavior.

A powerful time series model is the ARMA (Autoregressive Moving Average) process. The basic idea is that the series s_t at time t is affected by past values of s_t in a predictable manner. A general ARMA(p,q) can be written as:

$$\mathbf{s}_{t} = \alpha_{0} + \alpha_{1} \mathbf{s}_{t-1} + \dots + \alpha_{p} \mathbf{s}_{t-q} + \beta_{1} \mathbf{\varepsilon}_{t-1} + \dots + \beta_{p} \mathbf{\varepsilon}_{t-q} + \mathbf{\varepsilon}_{t}, \quad (A.2)$$

where ε_t is the prediction error at time t assumed to have a constant variance σ^2 . The terms with the α 's coefficients are the moving average terms. The terms with the β 's coefficients are the moving average terms.

In order for the ARMA model in (A.2) to have nice properties -i.e., to be stationary-, we need to check that the roots of the polinomial

1 - $(\alpha_1 z + \alpha_2 z^2 + ... + \alpha_p z^p) = 0$

lie outside the unit circle. In general, this requires that $|\alpha_1| < 1$.

The prediction error, ε_t , is just the difference between the realization of s_t and the prediction of s_t using the ARMA(p,q) model.

Example A.V.3: Suppose we estimate equation (A.2) and we obtain

$$s^{p}_{t} = a_{0} + a_{1} \, s_{t\text{-}1} + ... + a_{p} \, s_{t\text{-}q} + b_{1} \, \epsilon_{t\text{-}1} + ... + b_{p} \, \epsilon_{t\text{-}q},$$

where s^{p}_{t} is the predicted change in s_{t} , the a_{i} 's are the estimated α_{i} 's coefficients, and the b_{i} 's are the estimated β_{i} 's coefficients. Then,

 $\varepsilon_t = s_t - s^p_t$.

<u>Note</u>: Suppose that s_t represents changes in the MYR/USD exchange rate. According to (A.2), the past p changes in the MYR/USD exchange rate affect today's change in the MRY/USD exchange rate. Also, the past q prediction errors affect today's change in the MYR/USD exchange rate.

The key component of the ARMA model is to determine q and p. Several statistical packages provide identification tools to determine q and p. Many forecasters prefer to work with simpler AR(p) models. In this case, to determine p, a simple rule of thumb can be followed: start with an AR(1) model and add terms until the added terms are not statistically significant.

A.III Forecasting Using a Combination of Methods

Many forecasters use a combination of the methods described in A.I and A.II. The dependent variable might depend on theoretical grounds on a set of independent variables. On empirical grounds it has

been found that the dependent variable shows a high degree of autocorrelation. Although this autocorrelation is not present in the economic model, an economist might combine an economic model with an ARMA model to produce a better forecast.

Example A.V.4: Suppose a forecaster believes that changes in the monthly MYR/USD exchange rate are determined by the IFE. She also has found that an ARMA(1,1) helps to predict future changes in exchange rates. She decides to use the following forecasting model:

 $s_{MYR/USD,t,one-month} = \alpha_0 + \delta \ INT_t + \alpha_1 \ s_{t-1} + \beta_1 \ \epsilon_{t-1} + \epsilon_t,$

where ε_t is a prediction error with a constant variance, σ^2 .

A.IV Stationarity and Trends in Macroeconomic and Financial Data

In the previous sections, we have implicitly assumed that the dependent variable and independent variables are stationary. Roughly speaking, stationarity implies that the unconditional moments of a time series are independent of time. That is, they are constant.

Example A.V.5: the process for Y_t is said to be *weakly stationary* if:

$E[Y_t] = \mu$	for all t
$E[(Y_{t}-\mu)(Y_{t}-\mu)] = \sigma^{2}$	for $j = 0$
= 0	for $j \ge 0$.

The assumption of stationarity might not be appropriate for many of the economic and financial series used in practice. Several economic and financial series show clear trends: GDP, Consumption, CPI prices, stock prices, exchange rates, etc. For example, in Figure V.2, the CHF/USD shows a clear, predictable positive trend. This trend should be incorporated into any forecasting model.

There are two ways to achieve stationarity for these non-stationary series. The idea is to incorporate this trend in the model: (1) a deterministic time trend and (2) stochastic trend. The first model, also referred as *trend-stationary*, includes a deterministic time trend. The second model, also referred as a *unit root process*, uses first differences instead of levels.

Example A.V.6: Suppose y_t is a non-stationary series. (A) Trend-stationary process. $y_t = \alpha + \delta t + \varepsilon_t$,

where ε_t is a stationary error.

(B) Unit Root process. $y_t - y_{t-1} = \alpha + \varepsilon_t$,

where ε_t is a stationary error. This simple process is called a random walk with drift α .

We should note that this unit root process can be written in an AR(1) form:

 $y_t = \alpha_1 y_{t-1} + \alpha + \epsilon_t,$

where $\alpha_1 = 1$.

Both processes have different implications. If the series y_t follows a trend-stationary process, a shock has a temporary effect on the series, and the series eventually catches up with its trend. On the other hand, if y_t follows a unit root process, a shock might have permanent consequences for the level of future y_t 's.

There are several tests to check if a series has a unit root. These tests usually find a unit root on all major macroeconomic data. Of particular interest to us, exchange rates, GNP, money supply, and price levels have unit roots. Therefore, it is highly advisable to estimate models for these series in first differences.

It is common to take logs of the data before using it (see the Appendix of the Review Chapter). For small changes, the first difference of the log of a variable is approximately the same as the percentage change in the variable:

 $\log(y_t) - \log(y_{t-1}) = \log(y_t/y_{t-1}) = \log[1 + (y_t-y_{t-1})/y_t] \approx (y_t-y_{t-1})/y_t,$

where we have used the fact that for z close to zero, $log(1+z) \approx z$. It is usually convenient to multiply $log(y_t)$ by 100. Thus, the changes are measured in units of percentage change.

We should notice, however, that several economists claim that unit root tests are not very revealing. These economists claim that in finite samples -like the ones available to us- it is very difficult to distinguish between models with a unit root -i.e., α_1 =1- and stationary models with α_1 very close to 1.

Interesting readings:

Appendix V is based on **Introductory Econometrics with Applications**, by Ramu Ramanathan, published by Harcourt Brace Jovanovich.

Appendix V-B: Taylor Rules

According to the Taylor rule, the CB raises the target for the short-term interest rate, i_t , if: (1) inflation, I_t , raises above its desired level (2) output, y_t , is above "potential" output

The target level of inflation is positive (deflation is thought to be worse than positive inflation for the economy) The target level of the output deviation is 0, since output cannot permanently exceed "potential output."

John Taylor (1993) assumed the following reaction function by the CB:

 $i_t = I_t + \varphi (I_t - I_t^*) + \gamma y - gap_t + r^*$ (Equation BC.1)

where y-gap_t is the output gap –a percent deviation of actual real GDP from an estimate of its potential level-, and r^* is the equilibrium level or the real interest rate, which Taylor assumes equal to 2%. The coefficients φ and γ are weights, which can be estimated (though, Taylor assumes them equal to .5).

Let I_t^* and r^* in equation BC.1 be combined into one constant term, $\mu = r^* - \varphi I_t^*$. Then,

 $i_t = \mu + \lambda I_t + \gamma y\text{-gap}_t,$

where $\lambda = 1 + \varphi$.

For many countries, whose CB monitors S_t closely, the Taylor rule is expanded to include the real exchange rate, Rt:

 $i_t = \mu + \lambda \ I_t + \gamma \ y\text{-}gap_t + \delta \ R_t$

Estimating this equation for the US and a foreign country can give us a forecast for the interest rate differential, which can be used to forecast exchange rates.

Exercises:

1.- Go back to Example V.2.

- i. Take the autoregressive forecasting model, estimated above. What is $S^{F_{1997:4}}$?
- ii. Calculate the same forecast using the "naive" model.
- iii. Compare both forecasts with the in-sample PPP forecast.

2.- You work for a Tunisian investment bank. You have available the following quarterly interest rate series in the U.S., i_{USD}, and in Tunisia, i_{TND}, from 1998:4 to 1999:3 (TND=Tunisian Dinar). The TND/USD in 1998:4 is equal to 1.1646. Your job is to do quarterly out-of-sample forecasts of TND/USD exchange rate for the period 1999:2 1999:3, using the linear approximation to the International Fisher Effect (IFE).

Date	Tunisia	U.S.	Quarterly Forecast (S ^F)
1998:4	.0590	.0621	1.1646
1999:1	.0593	.0635	
1999:2	.0595	.0680	
1999:3	.0599	.0714	

i. Generate one-step-ahead forecasts -that is, as new information arrives, a new next period forecast is generated- for the period 1999:1-1999:4.

ii. Your firm uses the following forecasting regression model to forecast interest rates. Use a regression analysis.

$$\begin{split} i_{USD,t} &= .0075 + .93 \; i_{USD,t\text{-}1} + \epsilon_t. \\ i_{TND,t} &= .0060 + .97 \; i_{TND,t\text{-}1} + \epsilon_t. \end{split}$$

Generate out-of-sample forecasts for the period 1999:1-1999:4.

3.- Given that firms cannot forecast exchange rates, should they worry about currency risk?

4.- J. Cruyff, a Dutch designer company, wants estimate the monthly volatility of the weekly EUR/USD exchange rate. They use the following AR(1)-GARCH(1,1) model:

$$\begin{split} s_t &= [\log(S_t) - \log(S_{t-1})] x 100 = a_0 + a_1 s_{t-1} + e_{Ft}, \\ \sigma^2_t &= \alpha_0 + \alpha_1 \; e^2_{t-1} + \beta_1 \; \sigma^2_{t-1} + \; \delta e^2_{t-1} \; D_t, \end{split} \qquad \qquad e_t |\Psi_{t-1} \sim N(0, \sigma^2_t).$$

where Dt is the following dummy variable:

 $D_t = 0 \text{ if } e_{t-1} \ge 0$

 $D_t = 1$ if $e_{t-1} < 0$.

This GARCH model is an asymmetric model. Negative shocks increase the variance more than positive shocks. The persistence parameter should be redefined. That is, $\lambda = [\alpha_1 + \beta_1 + (1/2)\delta]$.

Using data from January 1974 till August 1997, the "quants" at J. Cruyff estimated the model for st:

(a) Find λ and calculate the unconditional variance, σ^2 . Is it well defined?

(b) Given that $e_{Aug 97} = -1.073$, and $\sigma^2_{Aug 97} = 7.436$, forecast the variance for September 1997.

(c) Forecast the variance for August 1998.

5.- You want to calculate the VAR of a position in EUR. The value of your position is USD 50 million. You estimated the volatility of changes in the USD/EUR exchange rates as 22%. The time interval is seven days. You use a 99% confidence interval to calculate VAR.