Finance 7397
Project 2

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1 Introduction

In this project, you will use various Monte Carlo techniques to value exotic options. Specifically, you will value an arithmetic Asian option and a Bermudan option.

2 European Call

1. Estimate using Monte Carlo the value of a call with a strike price of 100, a current stock price of 100, an interest rate of .01, a volatility of 40 percent, and a time to maturity of 6 months. Use 50,000 and 100,000 simulations.

2. Compare your estimates to the value given by the Black-Scholes formula.

3. Estimate the Delta of the option using both pathwise and likelihood ratio estimates.

4. For extra credit, use stratified sampling to simulate your terminal stock prices. Use 100 "buckets," each .01 in length. Calculate the value of the European call using the stock prices drawn using stratified sampling.

3 Asian Option Valuation

The Asian option is a call option with a strike price of \$100. The current price of the underlying stock is \$100. The annualized volatility is 35 percent. The interest rate is 5 percent. The option expires in 52 weeks. Every four weeks (every 28 days), the price of the underlying stock is determined. Call P_i the stock price at the end of month i, $i = 1, \ldots, 13$. At expiration, the 13 prices are averaged. The average, denoted by A, is:

$$A = \frac{1}{13}\sum_{i=1}^{13} P_i$$

At expiration the payoff to the option is $\max[A - K, 0]$, where $K = 100$.

1. Use Monte Carlo to value the option. Use the random number generator in Matlab or the computer program you are using to generate the simulation paths. Estimate the value of the option using (a) 13 prices per path, each 28/364ths of a year apart and 50,000 sample paths, and (b) 13 prices per path, and 100,000 sample paths. Create a table that gives the mean and mean squared error of the option values for each of your estimations.

2. Repeat the analysis from 2. above using the antithetic variate technique. Present your results in a table like that described above.

3. Although there is no closed form for an arithmetic Asian value, there is such a formula for a geometric Asian option. The payoff to the geometric Asian is $\max[G - K, 0]$, where

   $$G = (\Pi_{i=1}^{13} P_i)^{\frac{1}{13}}$$

   The formula is:

   $$V_G = e^{-r(T-t)}[e^{a+.5b}N(x) - KN(x - \sqrt{b})]$$

   where

   $$a = \ln(S_t) + \frac{1}{13}(r - .5\sigma^2) + \frac{(.5)(12)}{13}(r - .5\sigma^2)$$
   $$b = \sigma^2[\frac{1}{13} + \frac{(12)(25)}{(6)(13^2)}]$$

$$x = \frac{a - \ln K + b}{\sqrt{b}}$$

In this expression, $N(.)$ indicates the cumulative normal distribution. Use the normcdf function in Matlab to determine this.

4. Use Monte Carlo with a control variate technique to estimate the value of the arithmetic Asian. Use the geometric Asian as the control variate. Run the model using the two scenarios for the number of simulations and present your results (mean value and mean squared error) in a table. Present a scatter plot of the arithmetic values for each simulation against the geometric values for the corresponding simulation.

5. Estimate the Delta of the option using both pathwise and maximum likelihood methods.

4 Bermudan Option

A Bermudan option is similar to an American option in that it allows early exercise. However, whereas an American option allows exercise any time prior to expiration, the Bermudan can be exercised only at a limited number of dates prior to expiry.

Use the Longstaff-Schwartz method to value a Bermudan put. The put is on a stock with a current price of \$50 per share. The volatility (s) of the stock is 20 percent, annualized. The annualized continuously compounded interest rate (r) is 15 percent. The strike of the Bermudan is \$50 The Bermudan expires in 364 days. The put owner has the right to exercise the option every 28 days. There are therefore 13 exercise opportunities remaining.

For extra credit, modify the American put program you wrote for Project 1 to value the Bermudan. To make the problem somewhat easier, assume the exercise dates occur on the time step closest to the actual exercise date. The main modification necessary is that you only need to execute the projected successive over-relaxation step (in which you check for early exercise) at the time steps at which early exercise is possible. For the remaining time steps, just perform the matrix multi- plication you use in the program for a European put. That is, the finite difference Bermudan program combines elements of the American and European finite difference programs. How close is your Monte Carlo estimate to the finite difference estimate?