Computational Finance and Stochastic Calculus Homework 1

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- 1. Use Ito's Lemma to evaluate dX_t for the following functions, where W_t is a Brownian Motion:
 - $X_t = \exp\{-\int_0^t \lambda(s, W_s) dW_s .5 \int_0^t \lambda^2(s, W_s) ds\}$
 - $X_t = \exp\{.5t\}\sin(W_t)$
 - $X_t = (W_t + t) \exp\{-W_t .5t\}$
 - $X_t = \exp\{\mu t + \sigma W_t\}$
- 2. Unlike an option, a futures contract requires no initial investment. Use the classical Black-Scholes-Merton hedging derivation to show that the futures price $F(S_t, t)$ on a stock S_t (which satisfies the standard geometric Brownian motion stochastic differential equation) must satisfy the following PDE:

$$0 = F_t + rS_tF_S + .5\sigma^2 S_t^2 F_{SS}$$

What number of units of the stock (with a negative sign denoting a short sale) hedges a long position in a futures contract on a single share?