Finance 7397
Stochastic Calculus and Computational Finance
Project 1

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1 Introduction

In this project, you will utilize finite difference methods to value options on a stock. Specifically, you will value a European put option and an American put option. Remember that an American put can be exercised early, but the European cannot. You will also calculate the hedging parameters–the "Greeks"–of the options, i.e., their deltas, gammas, and thetas.

The dynamics for the stock price are:

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where dW_t is a Brownian motion. Thus, the log stock price dynamics are:

$$d\ln S_t = (\mu - .5\sigma^2)dt + \sigma dW_t$$

The current price of the stock underlying the options is \$100 per share. The volatility (σ) of the stock is 30 percent, annualized. The annualized continuously compounded interest rate (r) is 15 percent. The options expire in two months (i.e., $\tau = 1/6$).

2 Assignment Details

1. Write a program that uses the implicit algorithm to value a European put option on the stock struck at \$100 per share. (Note that your PDE

algorithm will determine the values for any stock price within the high and low stock prices on your grid.) Present a plot of the option value as a function of the current stock price; include the present value of the payoff to the option (i.e., $\max[e^{-rT}K - S, 0]$) in the same plot. **Hint.** Create a grid in the natural logarithm of the stock price. The high log stock price in your grid should be something like 4 standard deviations above the current log stock price. The low log stock price in your grid should be something like 4 standard deviations below the current log stock price. That is, center your grid around the current log stock price. Remember that your payoff at expiration is $\max[100 - exp(Z), 0]$, where Z is the natural logarithm of the stock price.) Also present a surface diagram that depicts the value of the option at all times and stock prices in your grid.

2. Value the option using 100 time steps and 250 stock price steps. Also value the option using 200 time steps and 250 stock price steps. Then use Richardson extrapolation to combine the two estimates to get a more accurate estimate.

3. Create a table that compares your estimates of the value of the European option with a strike price of \$100 produced by your finite difference algorithm to the values generated by the Black-Scholes formula. The table should include the values derived from your Richardson extrapolation calculations, and the Black-Scholes values, for the following stock prices: \$70, \$80, \$90, \$100, \$110, \$120, and \$130. **Hint:** You will probably have to interpolate to get the relevant values because your log stock price grid will not have points corresponding exactly to \$80, etc. If the values produced by your program are stored in a vector, you can use the interpolation features of MATLAB to find them. Alternatively, a simple linear interpolation is OK.

4. Value the option using 100 time steps and 500 stock price steps. Use Richardson extrapolation to combine the two estimates to get a more accurate estimate.

5. Calculate the current deltas, gammas, and thetas for the option. Present plots of these "Greeks" as functions of the stock price. Use the values derived from the Richardson extrapolation value based on 100 and 200 time step and 250 stock price runs. Remember that you have to adjust your Greeks to reflect the fact that your grid is in the log of the stock price but you want derivatives with respect to the stock price. That is, your program estimates $\partial V/\partial Z$ and $\partial^2 V/\partial Z^2$

but you want $\partial V/\partial S$ and $\partial^2 V/\partial S^2$.

6. Determine the value of the American put using the implicit method. Use projected successive overrelaxation. Present a plot of the option value as a function of the stock price; also include $\max[K-S, 0]$ in your plot. Value the option based on 100 time steps and 200 time steps, and 250 stock price steps; all plots should be based on the Richardson extrapolation value that combines these two valuations. Indicate the stock prices for which early exercise would be optimal today.

7. Calculate the current deltas, gammas, and thetas for the option. Present plots of these "Greeks" as functions of the stock price.

8. Create a program that values a European put option on this stock using an explicit finite difference method. First do the analysis assuming 50 time steps and 500 stock price steps. Are the results sensible? experiment with increasing the number of time steps to find the minimum number of time steps required for stability with 500 stock price steps, and then re-run the program with at least this many time steps.

Deliverables. You should create a document that includes: (a) the computer code used to complete the assignment; (b) a brief write-up of the results; (c) all of the tables and plots assigned as part of the project; and (d) a printed table of that includes the following columns: (i) the stock price (from lowest to highest in your grid–remember to convert the log stock prices to the actual stock price); (ii) the European option value corresponding to the stock prices; (iii) the present value of the European option payoff; (iv) the American option value corresponding to the stock prices; and (v) the value of the payoff to the American option.

I strongly suggest that you comment your computer code extensively so that it is easier for me to understand and evaluate.

Due date: 31 March, 2021. (Note this is one week later than indicted on the syllabus, to reflect the weather-related cancelation of one class session.)