Cornered: Market Power Manipulation by a Large Speculator

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1 Introduction

Market power manipulation–commonly known as a corner or squeeze–has bedeviled derivatives markets since their inception. The early histories of major exchanges, such as the Chicago Board of Trade, are filled with colorful stories of manipulation by larger than life characters. But manipulation is not just a historical curiosity. The soybean market was rocked by a major manipulation in 1989, copper was massively manipulated in the mid-1990s, Brent crude squeezed at various times in the 1990s and 2000, and recently there have been allegations of corners in aluminum and propane. Nor are corners limited to physical commodity markets. There is evidence of squeezes in US Treasury Bond futures in 1986 (Cornell and Shapiro, 1989); Salomon Brothers squeezed the Two-year Treasury Note market in 1991 (Jegadeesh, 1993); the UK Long Gilt contract was squeezed in the 1990s (Merrick, Naik, and Yadev, 2005); there are allegations that a corner of the Ten-Year Treasury Note futures contract (one of the world's largest) occurred in 2005; and the US Treasury and Federal Reserve have expressed serious concerns about chronic squeezes in the Treasury repo markets. There are also indications that credit derivatives have been squeezed in the aftermath of credit events. Allen, Litov, and Mei (2006) document a number of stock corners.

Manipulation casts a shadow over the markets even when corners are not occurring. Much of the regulatory apparatus in futures, options, and securities markets is directed at detecting, preventing, and deterring market power manipulation. Position limits, position reporting, market surveillance, internal compliance efforts, civil and criminal prosecution, and class action litigation are all directed at reducing the frequency and severity of manipulations. These resource-consuming efforts are driven by the fact that manipulation strikes at the very purpose of derivatives markets–risk transfer and price discovery–and imposes other deadweight losses in the form of distortions in production, consumption, and transportation.

Despite the importance of manipulation to the operation of derivatives markets, there are few models that predict: (a) how a manipulator can obtain market power from those that will suffer from it, (b) how the frequency, severity, and cost of manipulation varies with market conditions, (c) the welfare and distributive consequences of manipulation, and (d) what are the most efficient means to reduce the deadweight burdens of manipulation. Pirrong (1993) analyzes the "delivery end game" in detail, and Pirrong (2001) shows that market power manipulation can occur even in cash-settled futures contracts, but both of these articles start from the assumption that a trader has accumulated a large position, and do not analyze the trading process by which the trader can profitably accumulate this position.

The key problem in any analysis of how a large long accumulates a position that confers market power is: why do the shorts who suffer from a squeeze participate in the market? Allen, Litov, and Mei (2006) provide one approach to answering this question. In their model, arbitrageurs with information about fundamentals sometimes find it profitable to establish short positions even when there is a risk that a corner by an equally-well informed large trader who has sufficient resources to corner only some of the time; the profits from shorting when in possession of bearish information are large enough to offset the periodic losses from corners.

In an alternative approach to answering this question, Pirrong (1995a) builds on an insight of Kumar and Seppi (1992) to show that a large trader can use noise trader order flow to conceal partially his trading, and thereby accumulate a large position through the use of a randomized (mixed) trading strategy.

The manipulator in Pirrong (1995a)–as in Kumar-Seppi (1992)–is a liquidity demander. This article explores the possibility that a liquidity *supplier*–specifically, a large speculator–can corner a market. This is motivated by the fact that many famous manipulations have been undertaken by large speculative traders.

A straightforward modification of a standard Anderson-Danthine (1981) model of futures market equilibrium generates a variety of predictions regarding the factors that facilitate manipulation, its costs, and the costs and benefits of alternative means of reducing its frequency and severity. In this model, a large speculator and a competitive fringe of small speculators trade with agents who have an endowment that they can hedge by selling futures. The size of the hedger endowment can vary. Moreover, there is uncertainty about market fundamentals; supplies in the futures delivery market and non-delivery markets are random, and at the time they initiate their positions market participants know only the distribution of supply at expiration. The large speculator trades strategically, and depending on the size of the position he takes and the realization of the supply shock at contract expiration, sometimes squeezes the market at delivery.

Specifically, the model predicts:

- Manipulation occurs in equilibrium with positive probability in a derivatives market with a large speculative trader.

- Manipulation occurs more frequently, and is more severe, the larger the short hedging interest, the less elastic supply in the delivery market, the more elastic demand in the delivery market, the lower the risk tolerance of hedgers and small speculators, and the lower the risk tolerance of the competitive fringe of speculators.

- Manipulation reduces hedging effectiveness by reducing the correlation between the price of the futures and the prices of non-deliverables at expiration. This reduction in hedging effectiveness reduces the welfare of hedgers. The hedger welfare loss depends on the same factors that affect the frequency and severity of manipulation, and in the same way.

- Large speculators effectively obtain a manipulation option that they exercise when supply and demand conditions at expiration are favorable, but do not exercise under other market conditions.

- Non-manipulating speculators benefit from manipulation, but it harms hedgers.

- Speculative position limits reduce the frequency and severity of manipulation, but actually reduce welfare because they constrain the amount of risk that the large speculator absorbs from the hedgers. Although the large speculator exercises market power under some supply conditions, he also provides the beneficial function of taking on risk from the hedgers. Position limits impede this efficient risk transfer.

- In contrast, measures that deter the exercise of market power during the delivery period improve welfare because they constrain the exerice of market power at expiration without constraining the speculator's socially beneficial risk bearing function.

The model offers several advantages over Pirrong (1995), Kumar and Seppi (1992), and Allen, Litov, and Mei (2006). Specifically, it avoids the conventional contrivance of noise traders, and instead relies on a structural model of hedger participation. Moreover, it does not require that any trader (including the manipulator) possess superior information. Relatedly, the source of short selling–cross hedging–is realistic and pervasive in a variety of physical and financial markets, and is plausibly more important than information-based arbitrage trading. Also, the model does not rely on *ad hoc* random, exogenous wealth constraints on large trader positions to ensure the existence of an equilibrium with corners. Furthermore, it generates a variety of implications about how the frequency and severity of manipulation covaries with structural conditions (such as supply and demand conditions, and the amount of hedging activity); these, in turn, facilitate understanding of how the prevalence of corners should vary in the cross section and in time series.

The remainder of this article is organized as follows. Section 2 presents the model and derives some intermediate results. Section 3 sets out and discusses the main positive implications of the model. Section 4 discusses some policy implications. Section 5 summarizes the article.

2 The Model

2.1 The Physical Markets

There are $M+1$ physical markets for the commodity. There are no connections between the markets.[1] Demand in each market is $P = \theta_D - \phi_D q$, where θ_D and $\phi_D > 0$ are constants, P is the spot price in the market, and q is the quantity consumed. Supply in each market is $P = \tilde{\theta}_S + \phi_S q$, where $\phi_S > 0$ is a constant and q is the quantity supplied to the market. $\tilde{\theta}_S$ is a random variable. It takes the same value in each market, hence in the absence of derivatives trading, the spot prices of the commodity are perfectly correlated across markets. The density function of $\tilde{\theta}_s$ is $f(\tilde{\theta}_s)$, and its cumulative distribution function is $F(\tilde{\theta}_s)$. The density of $\tilde{\theta}_s$ has support $[0, \theta_D]$.[2]

2.2 The Futures Market

There is a futures contract traded on the commodity. The contract is settled by delivery in market 1.[3] Markets $2, \ldots, M+1$ are "out-of-position." Trading in the futures market occurs at time 0, and delivery occurs at time 1.

There are three types of participants in the futures market. First, there is a continuum of measure 1 of hedgers with endowments (contractual or physical) in each market $2, \ldots, M+1$. Hedger j at location $j \in [0,1]$ of the continuum has an endowment of y_j, and a risk tolerance (the inverse of his risk aversion

[1]This is for tractability. Introducing connections, through transportation, for instance, dramatically increases the complexity of the analysis. Despite this assumption, the simple model captures essential features of many markets. For instance, it is typically very inefficient to ship oil from, say, the Middle East to Sullom Voe (where delivery on Brent contracts occurs.) Thus, the Middle Eastern and Brent markets are effectively unconnected. Similarly, it is inefficient to ship soybeans from Brazil to Illinois for delivery against Chicago Board of Trade soybean contracts. The appendix presents a model that captures these conditions. In this model, there is a world consumption market for a commodity, and many dispersed production locations. There are no shipments between production locations. All of the implications presented in the main body of the text obtain for the model presented in the appendix.

[2]The assumptions of linearity and fixed demand facilitate the formal analysis, but the results presented below are not dependent on them. Along with the assumption of a uniform distribution of the supply shock, these assumptions permit closed form solutions for the expected spot price at expiration, the variance of the delivery market spot price, and the covariance between the delivery market and non-delivery market spot prices. Such closed forms are not available for other assumptions, e.g., isoleastic demand and lognormal distributions of supply and demand shocks. However, the model can be solved numerically under these more general assumptions. The results presented in section 3 below obtain under these alternative assumptions.

[3]Pirrong (2001) shows that the same results would obtain in a market where the futures are cash settled against the spot price in market 1.

coefficient) t_j. Hedgers are price takers in the physical and futures markets, and have mean-variance preferences. That is, hedger j chooses his futures position H_j to maximize:

$$U_{Hj} = y_j \bar{P}_c + H_j(\bar{P}_1 - F) - .5\frac{V_{Hj}}{t_j},$$

where $\bar{P}_c$ is the expected cash price in the hedger's market, $\bar{P}_1$ is the expected cash price in the delivery market at contract expiration, F is the futures price, and V_{Hj} is the variance of the hedger's wealth:

$$V_{Hj} = y_j^2 \sigma_c^2 + H_j^2 \sigma_1^2 + 2 y_j H_j \sigma_{1c},$$

where σ_c^2 is the variance of the cash price in markets $2, \ldots M+1$, σ_1^2 is the variance of the cash price in market 1, and σ_{1c} is the covariance between the prices in market 1 and markets $2, \ldots, M+1$.

The total endowment of the hedgers at each location is $Y = \int_{j \in [0,1]} y_j dj$. Hereafter I assume that hedgers are net long the commodity, so that $Y > 0$. The total hedger endowment is $MY > 0$.[4]

Second, there is a continuum of measure 1 of small speculators. Speculators have no endowment, and maximize mean-variance utilities. The risk tolerance of speculator i is t_i. Speculator i chooses her futures S_i position to maximize:

$$U_{Si} = S_i(\bar{P}_1 - F) - .5\frac{V_{Si}}{t_i}$$

where the variance of speculator wealth is:

$$V_{Si} = S_i^2 \sigma_1^2.$$

Since both hedgers and speculators are atomistic, they are price takers in the futures delivery process. Hence, they liquidate their entire futures positions at expiration.[5]

Third, and finally, there is a large speculator. This speculator has no endowment of the physical commodity, is risk neutral, and chooses a futures position x to maximize his expected wealth:

$$W = E\Pi(x, \tilde{\theta}_S) - xF$$

where $\Pi(x, \theta_S)$ is the large speculator's revenue at futures contract expiration, and the expectation is over the distribution of the supply shock $\tilde{\theta}_S$. This revenue

[4]This analysis assumes that there are no hedgers in the delivery market. This is for ease of exposition only. It is straightforward but tedious to incorporate hedging demand from the delivery market. This introduces heterogeneity among the hedgers (as out-of-position hedgers face basis risk, but those in the delivery market do not) which complicates the analysis, but does not change the fundamental results. Indeed, the presence of in-position hedgers facilitates the accumulation of large long speculative positions because these hedgers demands are not as sensitive to the speculator's trading because they do not suffer from basis risk.

[5]An atomistic trader's taking of delivery has no effect on the period 1 futures price, but the atomistic trader sells the commodity delivered to him at a price lower than he could sell his futures contract at expiration. Thus, he has no incentive to take delivery.

consists of two components: the revenue from sales of the commodity that is delivered to him, and revenue from the sales of futures contracts. Thus,

$$\Pi(x, \theta_S) = \max_Q \{Q(\theta_D - \phi_D Q) + (x - Q)(\tilde{\theta}_S + \phi_S Q)\} \tag{1}$$

In this expression, Q is the number of deliveries that the large speculator takes.[6] The first term in the bracketed expression is the revenues from deliveries. The long takes Q deliveries, which he sells at a spot price of $\tilde{\theta}_D - \phi_D Q$. The second term is the revenue from sales of futures contracts. If the long takes Q deliveries, he liquidates (sells) $x - Q$ futures contracts. At expiration, the futures price equals the marginal cost of delivery (Pirrong, 1993). For Q deliveries, this is $\tilde{\theta}_S + \phi_S Q$. If the solution to this maximization problem has $Q \leq Q_c$, where $Q_c = (\theta_D - \tilde{\theta}_S)/(\phi_D + \phi_S)$, the competitive quantity in the market, the manipulator sells both futures and whatever deliveries he takes at the competitive price $P_c = \tilde{\theta}_D - \phi_D Q_c = \tilde{\theta}_S + \phi_S Q_c$.

2.3 Equilibrium at Delivery

To squeeze, the speculator demands deliveries that exceed the competitive quantity in the delivery market (Pirrong, 1993). Therefore, the price in the delivery market at expiration (and hence the futures price at expiration) is higher than the competitive price if and only if $Q \geq Q_c$. By demanding excessive deliveries, the manipulator forces excessive production of the commodity, driving up the marginal cost of production; shorts must pay this inflated marginal cost of production in order to acquire the good for delivery. Immediately following expiration, the price in the delivery market falls below the competitive equilibrium price because the manipulator dumps the excessive supplies of the commodity in market 1. This post-delivery fall in price is referred to as the effect of "burying the corpse." The cost of burying the corpse (i.e., disposing the excessive deliveries at a depressed price) affects the profitability of manipulation, and the cornerer will take this effect into account when deciding how many contracts to liquidate and how many to close via delivery.

In this case, out-of-position hedgers suffer from an adverse move in the basis (i.e., the price at expiration rises relative to the prices in other markets), and there are deadweight losses arising from a distortion of production and consumption in the delivery market.

The delivery decision depends on $\tilde{\theta}_S$. Solution of the first order conditions for (1) implies:

$$Q = \frac{\theta_D - \tilde{\theta}_S + \phi_S x}{2(\phi_D + \phi_S)}$$

$Q \geq Q_c$ when:

$$\tilde{\theta}_S \geq \theta_S^* = \theta_D - \phi_S x$$

[6]For simplicity, I assume that the speculator has a long position. This occurs in equilibrium given the assumption about the endowment of hedgers.

That is, θ_S^* is the value of the supply shock such that a squeeze occurs if the supply curve's intercept is higher than this critical level. Note that deliveries are lower, the tighter are supply conditions (i.e., the larger is $\tilde{\theta}_S$). Further note that for a given x, the large speculator corners the market only when there is a sufficiently adverse supply shock. Moreover, since θ_S^* is decreasing in x, the speculator exercises market power more frequently (i.e., under a wider range of supply conditions), the larger is x. That is, the critical level of the supply curve is intercept is smaller with a larger x. This means that for larger x, squeezes occur for a wider range of supply shocks.

Since price in the delivery market at expiration depends on both x and $\tilde{\theta}_S$, the distribution of prices at expiration depends on x. When $\tilde{\theta}_S \leq \theta_S^*$ the price at expiration is the competitive price:

$$P_1 = \tilde{\theta}_S(1 - A) + \theta_D A$$

where $A = \phi_S/(\phi_D+\phi_S)$. When $\tilde{\theta}_S > \theta_S^*$, the delivery market price at expiration equals the marginal cost of producing Q units:

$$P_1 = \tilde{\theta}_S(1 - .5A) + .5\theta_D A + .5\phi_S Ax$$

Therefore, the expected price at expiration is:

$$\bar{P}_1 = \int_0^{\theta_S^*} [\tilde{\theta}_S(1-A)+\theta_D A] f(\tilde{\theta}_S) d\tilde{\theta}_S + \int_{\theta_S^*}^{\theta_D} [\tilde{\theta}_S(1-.5A)+.5\theta_D A+.5\phi_S Ax] f(\tilde{\theta}_S) d\tilde{\theta}_S$$

The expected price is increasing in x:

$$\frac{\partial \bar{P}_1}{\partial x} = .5\phi_S A[1 - F(\theta_S^*)] > 0$$

The variance of the market 1 price is:

$$\begin{aligned} \sigma_1^2 &= \int_0^{\theta_S^*} [\tilde{\theta}_S(1 - A) + \theta_D A]^2 f(\tilde{\theta}_S) d\tilde{\theta}_S \\ &+ \int_{\theta_S^*}^{\theta_D} [\tilde{\theta}_S(1 - .5A) + .5\theta_D A + .5\phi_S Ax]^2 f(\tilde{\theta}_S) d\tilde{\theta}_S - \bar{P}_1^2 \end{aligned} \tag{2}$$

The covariance between P_1 and P_c is:

$$\begin{aligned} \sigma_{1c} &= \int_0^{\theta_S^*} [\tilde{\theta}_S(1 - A) + \theta_D A]^2 f(\tilde{\theta}_S) d\tilde{\theta}_S \\ &+ \int_{\theta_S^*}^{\theta_D} [\tilde{\theta}_S(1 - A) + \theta_D A][\tilde{\theta}_S(1 - .5A) + .5\theta_D A + .5\phi_S Ax] f(\tilde{\theta}_S) d\tilde{\theta}_S \\ &- \bar{P}_1 \bar{P}_c \end{aligned} \tag{3}$$

Some results depend on the derivatives of the variance and covariance with respect to x. Note that:

$$\frac{\partial \sigma_1^2}{\partial x} = 2E(P_1 - \bar{P}_1)\frac{d(P_1 - \bar{P}_1)}{dx}.$$

For $\tilde{\theta}_S < \theta_S^*$,

$$\frac{d(P_1 - \bar{P}_1)}{dx} = -\hat{C}[1 - F(\theta_S^*)]$$

where

$$\hat{C} = .5\phi_S A x$$

is a constant.

For $\tilde{\theta}_S \geq \theta_s^*$,

$$\frac{d(P_1 - \bar{P}_1)}{dx} = \hat{C} - \hat{C}[1 - F(\theta_S^*)].$$

Therefore,

$$\begin{aligned} 2E(P_1 - \bar{P}_1)\frac{d(P_1 - \bar{P}_1)}{dx} &= 2\int_{\theta_S^*}^{\theta_D} (P_1 - \bar{P}_1)[\hat{C} - \hat{C}(1 - F(\theta_S^*))] f(\tilde{\theta}_S) d\tilde{\theta}_S \\ &- 2\int_0^{\theta_S^*} (P_1 - \bar{P}_1)\hat{C}[1 - F(\theta_S^*)] f(\tilde{\alpha}_s) d\tilde{\theta}_S \end{aligned}$$

where P_1 is a function of $\tilde{\theta}_S$. This can be rewritten as:

$$\begin{aligned} 2E(P_1 - \bar{P}_1)\frac{d(P_1 - \bar{P}_1)}{dx} &= 2\int_{\theta_S^*}^{\theta_D} (P_1 - \bar{P}_1)\hat{C} f(\tilde{\theta}_S) d\tilde{\theta}_S \\ &- 2\int_0^{\theta_D} (P_1 - \bar{P}_1)\hat{C}[1 - F(\theta_S^*)] f(\tilde{\theta}_S) d\tilde{\theta}_S \end{aligned}$$

By the definition of $\bar{P}_1$, the second term is zero, so:

$$\frac{\partial \sigma_1^2}{\partial x} = 2\hat{C}\int_{\theta_S^*}^{\theta_D} (P_1 - \bar{P}_1) f(\tilde{\theta}_S) d\tilde{\theta}_S > 0$$

A similar derivation implies that:

$$\frac{\partial \sigma_{1c}}{\partial x} = \hat{C}\int_{\theta_S^*}^{\theta_D} (P_c - \bar{P}_c) f(\tilde{\theta}_S) d\tilde{\theta}_S > 0$$

Now note that:

$$\int_{\theta_S^*}^{\theta_D} (P_1 - \bar{P}_1) f(\tilde{\theta}_S) d\tilde{\theta}_S = \int_{\theta_S^*}^{\theta_D} P_1 f(\tilde{\theta}_S) d\tilde{\theta}_S - \bar{P}_1[1 - F(\theta_S^*)]$$

and

$$\int_{\theta_S^*}^{\theta_D} (P_c - \bar{P}_c) f(\tilde{\theta}_S) d\tilde{\theta}_S = \int_{\theta_S^*}^{\theta_D} P_c f(\tilde{\theta}_S) d\tilde{\theta}_S - \bar{P}_c[1 - F(\theta_S^*)]$$

Therefore,

$$\hat{C}\int_{\theta_S^*}^{\theta_D} (P_1 - \bar{P}_1) f(\tilde{\theta}_S) d\tilde{\theta}_S > \hat{C}\int_{\theta_S^*}^{\theta_D} (P_c - \bar{P}_c) f(\tilde{\theta}_S) d\tilde{\theta}_S$$

if

$$\int_{\theta_S^*}^{\theta_D} P_1 f(\tilde{\theta}_S)d\tilde{\theta}_S - \bar{P}_1[1-F(\theta_S^*)] > \int_{\theta_S^*}^{\theta_D} P_c f(\tilde{\theta}_S)d\tilde{\theta}_S - \bar{P}_c[1-F(\theta_S^*)].$$

This holds if:

$$\int_{\theta_S^*}^{\theta_D} (P_1 - P_c)\frac{f(\tilde{\theta}_S)}{1-F(\theta_S^*)}d\tilde{\theta}_S > \bar{P}_1 - \bar{P}_c$$

The left-hand-side is the expected value of the difference between the manipulative and competitive prices conditional on manipulation occuring, and the right-hand-side is the unconditional mean of the difference between the manipulated price and the price in the markets where manipulation does not occur. Since the market 1 price and the price in the other markets is the same when manipulation does not occur, but the market 1 price is higher when manipulation occurs, the conditional expectation of the difference must exceed the unconditional expectation of the difference. This, in turn, implies that

$$\frac{\partial \sigma_1^2}{\partial x} \geq 2\frac{\partial \sigma_{1c}}{\partial x}.$$

This further implies that due to manipulation, the correlation between the market 1 price and the prices in the out-of-position markets declines as the size of the large speculator's position increases over some range of x.

2.4 Futures Market Equilibrium

Information in the model is perfect and complete. Hedgers and the small speculator observe the large speculator's trade, x, so the results are not driven by the ability of the large speculator to hide in the order flow. Moreover, all market participants observe Y, and have the same information about the distribution of $\tilde{\theta}_S$. Hence, manipulation does not occur due to any information advantage by the large speculator.

It is well known that the price-taking, mean-variance utility out-of-position hedger j chooses:

$$H_j = -y_j\frac{\sigma_{1c}}{\sigma_1^2} + t_j\frac{\bar{P}_1 - F}{\sigma_1^2}.$$

The first term is the variance minimizing hedge. The second term is the speculative component of the hedger's position. Short hedging is costly when there is a risk premium ($\bar{P}_1 - F > 0$.) The second term reflects the adjustment to the hedge arising from this risk premium. Similarly, speculator i chooses:

$$S_i = t_i\frac{\bar{P}_1 - F}{\sigma_1^2}.$$

Equilibrium in the futures market requires:

$$x + M\int_{j\in[0,1]} H_j dj + \int_{i\in[0,1]} S_i di = 0.$$

Combined with the expressions for H_j and S_i, this implies that in equilibrium:

$$x - MY\frac{\sigma_{1c}}{\sigma_1^2} + (MT_H + T_S)\frac{\bar{P}_1 - F}{\sigma_1^2} = 0$$

where $T_H = \int_{j\in[0,1]} t_j dj$ and $T_S = \int_{i\in[0,1]} t_i di$. Thus:

$$F = \bar{P}_1 - MY\frac{\sigma_{1c}}{T_T} + x\frac{\sigma_1^2}{T_T}, \tag{4}$$

where $T_T = MT_H + T_S$ is the total risk bearing capacity (risk tolerance) supplied by agents other than the large speculator.

Note that

$$\frac{dF}{dx} = \frac{\partial \bar{P}_1}{\partial x} - \frac{MY}{T_T}\frac{\partial \sigma_{1c}}{\partial x} + \frac{\sigma_1^2}{T_T} + \frac{x}{T_T}\frac{\partial \sigma_1^2}{\partial x}. \tag{5}$$

The large speculator takes into account this impact of his trading activity on the futures price when choosing x. Specifically, when choosing x to maximize (1), he solves the first order condition:

$$\frac{dW}{dx} = E\frac{d\Pi(x,\tilde{\theta}_S)}{dx} - F - x\frac{dF}{dx} = 0.$$

The envelope implies that $d\Pi(x,\tilde{\theta}_S)/dx = \bar{P}_1$. Substituting from (5) and (6), the first order condition becomes:

$$\frac{xMY}{T_T}\frac{\partial \sigma_{1c}}{\partial x} + \frac{MY}{T_T}\sigma_{1c} - x\frac{\partial \bar{P}_1}{\partial x} - \frac{2\sigma_1^2 x}{T_T} - \frac{x^2}{T_T}\frac{\partial \sigma_1^2}{\partial x} = 0. \tag{6}$$

3 Analysis of the Futures Market Equilibrium

The model makes several predictions about equilibrium and welfare in a market that can be manipulated. Some of these results can be proved analytically. Others can be shown numerically.

The key analytical results are:

- Manipulation occurs with positive probability. That is, $Q \geq Q_c(\tilde{\theta}_S)$ with positive probability. To see why, note that manipulation occurs with positive probability when $\theta_S^* < \theta_D$. This occurs, in turn, whenever $x > 0$. Consider the first derivative of the manipulator's objective function W when $x = 0$:

  $$\frac{dW}{dx} = \frac{MY}{T_T}\sigma_{1c} > 0$$

  Thus, $x = 0$ is not an equilibrium, and in fact the large speculator chooses $x > 0$.

- The foregoing implies that stealth is not necessary for corners to occur in equilibrium.[7] Although the ability to trade without being detected (or at least with some possibility of escaping detection) presumably reduces the price impact of the large speculator's trading, and thereby increases the profitability of that trading, in the model the large speculator squeezes with positive probability even though his trading is completely transparent. Nonetheless, corners are not perfectly predictable at the time traders initiate positions, because supply shock realizations also determine whether or not they occur.

- The frequency of manipulation, and the deadweight losses arising therefrom, are increasing in Y. Note that

$$dx/d(MY) = -\frac{\partial^2 W/\partial x \partial(MY)}{\partial^2 W/\partial x^2}.$$

  The numerator in this expression is

$$\frac{x}{T_T}\frac{\partial \sigma_{1c}}{\partial x} + \frac{\sigma_{1c}}{T_T} > 0,$$

  but the denominator is negative (by the second order conditions for a maximum), so the derivative is positive. This occurs because the large speculator takes a larger position when the hedging interest is larger. Since the severity and frequency of manipulation are increasing in x, the greater the hedging demand for futures, the worse the deadweight losses from corners. This implies that futures markets with a large out-of-position hedging interests are more susceptible to manipulation than smaller futures markets.

- The frequency, severity, and cost of manipulation is decreasing in hedger and small speculator risk tolerance T_T. Note that:

$$\frac{dx}{dT_T} = -\frac{\partial^2 W/\partial x \partial T_T}{\partial^2 W/\partial x^2}$$

  Moreover:

$$\frac{\partial^2 W}{\partial x \partial T_T} = \frac{-1}{T_T}[-\frac{2\sigma_1^2 x}{T_T} - \frac{x^2}{T_T}\frac{\partial \sigma_1^2}{\partial x} + \frac{xMY}{T_T}\frac{\partial \sigma_{1c}}{\partial x} + \frac{MY}{T_T}\sigma_{1c}]$$

  By (6), term in brackets is $x\partial \bar{P}_1/\partial x > 0$, so $\partial^2 W/\partial x \partial T_T$ is negative, and $\partial^2 W/\partial x^2$ is negative from the second order conditions for a maximum. Thus, dx/dT_T is negative. Since manipulation is more frequent and severe, the larger is x, this implies that greater risk bearing capacity by small speculators and hedgers reduces manipulation.

[7]Easterbrook (1986) emphasizes the importance of position concealment on the success of manipulation. The manipulator in Pirrong (1995a) and Kumar-Seppi (1992) also profits from the ability to conceal positions.

This is an intuitive result. The large speculator faces a lower and more elastic derived demand for his services, the greater the risk tolerance of the hedgers, and the greater the risk tolerance of the small speculators he must compete with. With a lower and more elastic derived demand, he chooses a smaller position. Hence, greater competition for the large speculator reduces the severity of manipulation problems.

- The large speculator chooses a smaller position x when he can manipulate than when manipulation is precluded (due, for instance, to the imposition of punishment that deters this conduct.) Note that when manipulation is precluded, the large speculator buys $.5MY$ contracts because in this case, the average price in the delivery market, the covariance between prices in the delivery and non-delivery markets, and the variance of the price in the delivery market do not depend on x (because in the absence of manipulation, price in the delivery market is the competitive price regardless of the size of the large speculator's position.) Thus, the relevant first order condition is:

$$\frac{dW}{dx} = \frac{MY}{T_T}\sigma_{1c} - \frac{2\sigma_1^2 x}{T_T} = 0$$

  which implies $x = .5MY$ since $\sigma_1^2 = \sigma_{1c}$ in the absence of manipulation. Now consider the derivative of the speculator's objective function evaluated at $x = .5MY$ when manipulation can occur:

$$\frac{dW}{dx} = \frac{M^2Y^2}{2T_T}[\frac{\partial \sigma_{1c}}{\partial x} - .5\frac{\partial \sigma_1^2}{\partial x}] - \frac{Y}{T_T}\frac{\partial \bar{P}_1}{\partial x} + \frac{Y}{T_T}[\sigma_{1c} - \sigma_1^2].$$

  Since each of these terms is negative when $x > 0$, the derivative is negative, meaning that the large speculator increases his wealth by reducing his futures position below $.5MY$.

  This result may seem counterintuitive, but it is sensible. The prospect for manipulation affects prices. When manipulation is possible, the futures price is higher, and more sensitive to the large speculator's purchases than when it is not. Recognizing that his purchases drive up prices, the large speculator trades less intensively.

- Hedgers trade less when manipulation can occur. That is, the $|H_j|$ are smaller when corners occur than when they are precluded.

  This can be proved by contradiction. Recall that

$$-H_j = y_j\frac{\sigma_{1c}}{\sigma_1^2} - \frac{\bar{P}_1 - F}{\sigma_1^2}$$

  Since when corners can occur $\sigma_{1c}/\sigma_1^2 < 1$, whereas this ratio equals 1 when corners cannot occur, the only way that $|H_j|$ can increase when corners can occur is for $-(\bar{P}_1 - F)/\sigma_1^2$ to increase, and hence for $(\bar{P}_1 - F)/\sigma_1^2$ to decline. If this happens, however, S_i declines for all i. Furthermore, it was just shown that x is smaller when the large speculator can corner than when he cannot. Thus, for the $|H_j|$ to be higher when manipulation can occur than when it cannot, the $\int_{i\in[0,1]} S_i di + x$ must be smaller. This violates the equilibrium condition.

$$-M \int_{j\in[0,1]} H_j dj = \int_{i\in[0,1]} S_i di + x$$

Derivation of results regarding the impact of manipulation on welfare and risk premia must be performed numerically because expression (6) is in general a highly non-linear function of x, and cannot be solved in closed form. This numerical solution provides insights on the impact of manipulation on risk premia and futures pricing. Several figures depict visually the key results.

The numerical solution assumes that the distribution of $\tilde{\theta}_S$ is uniform on the interval $[0, \theta_D]$. In this case:

$$\bar{P}_1 = \frac{1}{\theta_D}\{\int_0^{\theta_S^*} [\tilde{\theta}_S(1-A) + \theta_D A] d\tilde{\theta}_S + \int_{\theta_S^*}^{\theta_D} [\tilde{\theta}_S(1-.5A) + .5\theta_D A + .5\phi_S A x] d\tilde{\theta}_S\}$$

Simplifying this integral, it is straightforward to show that $\bar{P}_1$ is an increasing, quadratic function of x.

The variance of the market 1 price is:

$$\begin{aligned} \sigma_1^2 &= \frac{1}{\theta_D}\{\int_0^{\theta_S^*} [\tilde{\theta}_S(1-A) + \theta_D A]^2 d\tilde{\theta}_S \\ &+ \int_{\theta_S^*}^{\theta_D} [\tilde{\theta}_S(1-.5A) + .5\theta_D A + .5\phi_S A x]^2 d\tilde{\theta}_S\} - \bar{P}_1^2 \qquad (7) \end{aligned}$$

Simplification of this integral implies that this variance is a cubic function of the large speculator's position x.

The covariance between P_1 and P_c is:

$$\begin{aligned} \sigma_{1c} &= \frac{1}{\theta_D}\{\int_0^{\theta_S^*} [\tilde{\theta}_S(1-A) + \theta_D A]^2 d\tilde{\theta}_S \qquad (8) \\ &+ \int_{\theta_S^*}^{\theta_D} [\tilde{\theta}_S(1-A) + \theta_D A][\tilde{\theta}_S(1-.5A) + .5\theta_D A + .5\phi_S A x] d\tilde{\theta}_S\} \\ &- \bar{P}_1 \bar{P}_c \end{aligned}$$

This is also a cubic function of x. Along with (6) these results imply that the first order condition is a quartic function of x.

The figures are are derived from the solution of the problem for $\theta_D = 200/3$, $\theta_D = 1$, $\phi_D = 1$, $\phi_S = .25$, and $M = 10$. Moreover, T_T is set so that in the absence of the large speculator, when Y is equal to the mean value of Q_c, $\bar{P}_c - F = .025\bar{P}_c$, that is, so that the mean futures price discount (which rewards speculators for bearing risk) is 2.5 percent of the mean competitive price. Moreover, $T_H = .1T_T$.

- Although the likelihood of a squeeze is higher when x is large, and x is increasing in Y, the futures price is decreasing in Y. Increasing Y has two effects: a hedging pressure effect, and a manipulation effect. When Y is large, hedgers want to sell more futures, *ceteris paribus.* This tends to depress the futures price. When corners can occur, (a) the effect of manipulation on hedging effectiveness reduces hedger supply of futures positions, and (b) increases the expected spot price at expiration; these factors tend to counter the impact of greater hedging pressure. In equilibrium, however, the hedging pressure effect dominates, and therefore the amount of risk that hedgers transfer to small speculators is increasing in Y. This increase in hedging pressure depresses the futures price. Figure 1 presents the relation between the futures price and Y.

  This result suggests that a manipulator's trading will exhibit *negative* correlation with prices. If Y varies over time, the large speculator's purchases will be largest when Y is largest. The resultant hedging pressure causes the futures price to be lower than with smaller Y. Thus, the large speculator's purchases are negatively correlated with the futures price. Moreover, when he squeezes, his sales occur at a high price.

  This result also implies that the risk premium $\bar{P}_1 - F$ is increasing in Y because (a) $\bar{P}_1$ is increasing in x, which is increasing in Y, and (b) F is decreasing in Y.

- Total surplus (hedger utility plus small speculator utility plus large trader expected profit minus deadweight loss from consumption and production distortions in the delivery market) as a fraction of total surplus when manipulation is precluded is declining in Y (M). This is depicted in Figure 2. This occurs because as Y increases and manipulation becomes more frequent and severe, hedgers must incur more basis risk. In response, hedgers sell fewer futures per unit of endowment because the greater frequency and severity of manipulation reduces σ_{1c}/σ_1^2 (the variance minimizing hedge ratio), and inflates the risk premium $(\bar{P}_1 - F)/\sigma_1^2$. That is, manipulation reduces the effectivness of hedging (by increasing the variability of the basis), and also increases its cost (the risk premium.) This reduces hedger utility. Speculator utility rises, but not by enough to offset the effect of (a) the deadweight losses arising from manipulation, and (b) the decrease in hedger utility.

- The frequency, severity, and cost of manipulation is increasing in ϕ_S. That is, manipulation is less (more) frequent, and severe, and costly, when supply in the delivery market is more (less) elastic. This is readily understood. A higher ϕ_S (i.e., a flatter supply curve) means that (a) the competitive quantity in the delivery market is larger, *ceteris paribus,* so a larger position is required to manipulate the market, and (b) it is less costly to enhance supply in the delivery market in response to a manipulator's demand for excessive deliveries. Similarly, the frequency, severity, and cost of manipulation is decreasing in ϕ_D. When demand is inelastic, the cost of burying the corpse is large because the less elastic the demand, the more the cornerer depresses the post-delivery price when he dumps his deliveries on the market. The higher cost of burying the corpse reduces the speculator's incentive to manipulate the market.

4 Policy Implications

This analysis has some important policy implications. First, restrictions on the size of the position that large speculator can hold actually reduce welfare. Figure 3 depicts the ratio between total surplus when the large long can only hold half as many futures contracts as he would buy in the absence of a position limit, and total surplus when the large long's positions are not restricted, as a function of Y. Note that this fraction is less than or equal to one, and decreasing in Y. This is an important result, because position limits are a widely employed (or advocated) as a means of preventing manipulation.[8]

This result reflects the fact that the large speculator plays two roles, one beneficial, one not. The speculator's rent-seeking manipulative activities reduce surplus. However, the speculator also facilitates hedging by bearing risk more efficiently than the hedgers, or the small speculators. Although position limits reduce manipulation, and the deadweight losses associated therewith, they also limit the amount of risk that the large speculator absorbs from the hedgers. In this model, the losses arising from this latter effect of position limits more than offset the benefits arising from a reduction in manipulation. That is, position limits throw out the baby with the bath. They indiscriminately curtail the large speculator's efficiency enhancing activities as well as his efficiency reducing ones. Indeed, since as noted above a speculator who can manipulate buys fewer futures than he would than one who is precluded from doing so, position limits are perverse because they restrain his trading–and risk bearing–even further.[9]

This implies that different measures to curb manipulation are preferable to speculative position limits. It further implies that a desirable policy is one that constrains the ability to exercise market power at delivery without constraining the large speculator's socially beneficial risk bearing function. That is, efforts forcused on reducing the exercise of market power at expiration are more efficient than restrictions on the accumulation of large positions because the former measures constrain rent seeking activities without limiting the large speculator's ability to perform the valuable function of absorbing risk from hedgers.[10]

[8] A different form of position limit also reduces welfare below the level that would obtain when manipulation can occur and the large speculator's positions are not restricted. Specifically, welfare falls if the manipulator's position is limited to the average competitive supply in the delivery market.

[9] Since x is less than half the position the large speculator would amass if manipulation is precluded, this position limit forces him to hold one-quarter of the position he would hold if he were prevented from squeezing at expiration.

[10] Implementation of measures that preclude the exercise of market power at expiration will not result in a first best outcome. The large speculator still exercises market power in period 0 because he faces a downward sloping demand for his risk bearing services (due to the limited risk bearing capacity of the hedgers and small speculators.) As a result he buys

Another way to reduce manipulative losses without constraining speculative risk bearing is to deter the exercise of market power through the imposition of sanctions *ex post.* The analysis also provides insight on this issue. Specifically, the model implies that the large speculator essentially holds a manipulation option. For a given x, the large speculator's delivery period revenues are an increasing, convex function of the supply shifter $\tilde{\theta}_S$. The convexity arises because $\Pi(x, \tilde{\theta}_S)$ is a convex function of $\tilde{\theta}_S$. Note that, by the envelope theorem:

$$\frac{d\Pi(x, \tilde{\theta}_S)}{d\tilde{\theta}_S} = x - Q$$

Therefore:

$$\frac{d^2\Pi(x, \tilde{\theta}_S)}{d\tilde{\theta}_S^2} = -\frac{dQ}{d\tilde{\theta}_S} = \frac{1}{2(\phi_D + \phi_S)} > 0$$

Intuitively, the manipulator takes fewer deliveries when supply is lower because he profits on the contracts he liquidates and loses money on the deliveries he takes due to the cost of burying the corpse. Low supply allows him to liquidate more contracts at a higher price and reduce the losses from burying the corpse. Thus, $d^2\Pi(x, \tilde{\theta}_S)/d\tilde{\theta}_S^2 > 0$.

This convexity means that the large speculator's payoff at expiration is effectively an option on the supply shock. For small values of the supply shifter (indicating abundant supply), the speculator does not exercise market power. Only when supply conditions are sufficiently tight (i.e., when $\tilde{\theta}_S > \theta_S^*$) does he squeeze, and due to the convexity, he squeezes harder, the lower is supply.

This has important policy implications because of some decisions in manipulation cases. In particular, in *in re Indiana Farm Bureau,* the Commodity Futures Trading Commission found that a large trader could not be found guilty of manipulation if he acquired his futures position without the specific intent to manipulate the market, but just took advantage of tight market conditions at delivery that were not forseen at the time he initiated his position.[11] An administrative law judge made a similar determination in *in re Abrams.*[12] Relatedly, it is sometimes said that markets are subject to "natural squeezes" that occur because of adverse supply shocks during the delivery period (e.g., a crop failure or transportation breakdown at delivery.)[13]

The model suggests that these decisions, and the idea of a "natural squeeze," are logically defective. In particular, the model implies that the large speculator

fewer futures contracts than is first best to inflate the risk premium. Recall that if he cannot manipulate, the large speculator purchases futures in a quantity equal to half the endowment of the hedgers. In the first best solution, the large speculator should purchase futures in an amount equal to 100 percent of the hedger endowment.

[11] In re Indiana Farm Bureau Cooperative Association [1982-1984 Transfer Binder] Commodity Futures Law Reporter (CCH) ¶21,796 (CFTC, 1982).

[12] In re Abrams [1988-1989 Transfer Binder] 2 Commodity Futures Law Reporter (CCH) ¶24,408 (CFTC, 1989).

[13] This concept has been widely accepted at least since the time of the *Report of the Federal Trade Commission on the Grain Trade* (1926). Some legal commentary similary endorses this view. Pirrong (1997) criticizes it.

has no specific intent to manipulate the market when he *initiates* his position. He takes a position that permits him to manipulate if conditions are favorable. That is, in this model, all squeezes are natural in some sense in that whether or not the market is squeezed depends on exogenous supply conditions unknown to the speculator when he initiates his position. Put another way, in the model squeezes occur, and are most severe, when supply conditions are tight due to factors beyond the speculator's control. He exploits these conditions opportunistically. The logic of *Indiana Farm Bureau* gives the speculator a legal pass on this conduct.

The model also sheds light on the political economy of anti-manipulation restrictions. Although manipulation causes deadweight losses, it also has distributive effects. The utility of hedgers is lower when manipulation can occur, but in the numerical solution of the model the utility of the small speculators is actually higher. This reflects the finding that manipulation actually inflates the risk premium. Relatedly, it occurs because in equilibrium, a large speculator who can squeeze buys fewer futures than one who cannot (due, for instance, to a limit on the number of deliveries he can take.) This effectively increases the derived demand for the small speculators' risk bearing services. Although the reduction in hedging reduces this derived demand, in the model the former effect dominates, thereby increasing the small speculators' utilities.

Pirrong (1995b) argues that these distributive effects explain the historical reluctance of exchanges to adopt anti-corner rules (or to enforce them aggressively). The model provides support for that view, in that exchange memberships were dominated by small speculators and brokers that serviced them.

5 Summary and Conclusion

This article presents a straightforward model of derivatives market manipulation by a large speculator. In contrast with earlier models of manipulation by liquidity demanders, this model shows that large liquidity *suppliers* sometimes manipulate markets at contract expiration in equilibrium even if their trading is not partially or fully concealed. The model implies that these periodic corners reduce the hedging effectiveness of derivatives markets, inflate risk premia, and transfer wealth from hedgers to speculators. Hedgers use manipulated contracts (albeit less intensively) because they still allow them to reduce risk.

The model suggests that certain regulatory tools–notably speculative position limits–and legal doctrines–particularly the requirement that a manipulation conviction requires a finding of specific manipulative intent at the time a position is initiated and the concept of a "natural squeeze"–are perverse. Speculative position limits indeed restrict speculators' abilities to exercise market power, but they also inefficiently constrain their beneficial risk bearing activities, and in the model the latter effect dominates. The model also implies that a large speculator's behavior at contract expiration depends not merely on the size of his position, but on the supply and demand conditions prevailing at expiration. The large speculator accumulates a position knowing that he will manipulate only under certain states of the world during the delivery period. The speculator holds a manipulation option which he may, or may not, exercise, depending on the realization of supply and demand conditions at expiry. That is, all squeezes are in some sense "natural," because exogenous supply and demand conditions determine whether the holder of a large long position can profitably manipulate.

The model also shows that the size of the hedging interest–parameterized in the model by the size of hedger endowments–is a crucial determinant of the frequency and severity of manipulation. Manipulation problems are more acute, the larger the short out-of-position hedging interest. The models suggests that the recent growth in the size of many derivatives markets, fueled by the growth of hedge funds and trends in global savings, has made these markets more vulnerable to corners and squeezes.

The model further suggests that structural features in derivatives markets encourage manipulation. In particular, liquidity considerations often make it efficient to concentrate trade in a single contract, or a small number of contracts. For instance, global oil hedging is dominated by the Brent and West Texas Intermediate contracts. The deliverable supply in these markets is small relative to hedging interest. That is, most oil hedgers are "out-of-position" because they are not hedging Brent or WTI cash market positions. The model implies that these markets are particularly vulnerable to manipulation. The large hedging interest facilitates the accumulation of large speculative positions; the limited sizes of the delivery markets (relative to hedging interest and speculative positions) makes these contracts vulnerable to squeezes by the holders of these large speculative positions.

In sum, manipulation occurs in a standard futures market equilibrium model even when information about trading activity is known to all. Although stealth arguably facilitates manipulation, it is not necessary for it to occur. Instead, the frequency and severity of manipulation is driven more by structural market conditions, notably the size of the out-of-position hedging interest relative to the size of the delivery market. These manipulations are costly, impeding efficient risk transfer and causing deadweight distortions in consumption and production. Thus, well-designed regulations that constrain the ability of large speculators to exercise market power at contract expiration can improve welfare. But the model also shows that some commonly used regulations can actually make matters worse.

A Appendix: Models With Demand and Supply Connections Between Markets

This appendix presents an alternative model demonstrate that the results derived above hold under alternative assumptions about the structure of the physical markets. The motivation for this model is a global market for a commodity, such as oil or corn. In such markets, there is a global demand for the commodity, and production occurs in many spatially dispersed areas. Typically, the producing regions ship to consumption markets, and there are no shipments between production regions. For instance, soybeans are produced in South America and the United States, and shipped to major consumption markets in Asia and Europe, but soybeans are not shipped from South America to the American Midwest. Similarly, Brent crude oil and WTI crude are shipped to major consumption markets, but recently Brent has not been shipped to the US, or WTI to Britain.

The "world" demand for the commodity in the model is $Q_d = \alpha_d - \beta_d P$ where P is the price of the good in the consumption market, Q_d is total consumption, and $\alpha_d > 0$ and $\beta_d > 0$ are parameters. There are $M+1$ production areas for the commodity. The supply curve in each market is $Q_s = \tilde{\alpha}_s + \beta_s P$ with $\beta_s > 0$, and $\tilde{\alpha}_s > \underline{\alpha}_s = -\alpha_d \beta_s / \beta_d$.[14] As in the main text, $\tilde{\alpha}_s$ is the same in each production location. Also as in the main text, there is a futures contract with delivery in market 1. Finally, $\tilde{\alpha}_s$ is distributed uniform on the interval $[\underline{\alpha}_s, \bar{\alpha}_s]$.

In the absence of manipulation, the competitive price is:

$$P_c = \frac{\alpha_d - (M+1)\tilde{\alpha}_s}{\beta_d + (M+1)\beta_s}$$

The competitive quantity is:

$$Q_c = \frac{\alpha_d \beta_s + \tilde{\alpha}_s \beta_s}{\beta_d + (M+1)\beta_s}.$$

If the large speculator takes Q deliveries, the price in the consumption market (at which the speculator sells the units delivered to him) is determined by the solution of the equilibrium condition:

$$M(\tilde{\alpha}_s + \beta_s P) + Q = \alpha_d - \beta_d P,$$

which implies:

$$P = \frac{\alpha_d - M\tilde{\alpha}_s - Q}{\beta_d + M\beta_s}.$$

Moreover, the marginal cost of delivery of Q units in market 1 is:

$$P_1 = -\frac{\tilde{\alpha}_s}{\beta_s} + \frac{Q}{\beta_s}.$$

Therefore, the analog to the manipulator's maximization problem (1) in this model is:

$$\Pi(\alpha_s, x) = \max_Q \{Q \frac{\alpha_d - M\tilde{\alpha}_s - Q}{\beta_d + M\beta_s} + (x - Q)(-\frac{\tilde{\alpha}_s}{\beta_s} + \frac{Q}{\beta_s})\}.$$

Solution to this problem implies that the large speculator holding $x > 0$ futures chooses to take deliveries:

$$Q = \frac{\alpha_d \beta_s + \tilde{\alpha}_s \beta_d + x(\beta_d + M\beta_s)}{2[\beta_d + (M+1)\beta_s]}.$$

[14] This constraint ensures that the competitive quantity is always positive.

$Q \geq Q_c$ when:

$$\tilde{\alpha}_s \leq \alpha_s^* := \frac{-\alpha_d \beta_s}{\beta_d} + \frac{x(\beta_d + M\beta_s)}{2\beta_d(M + .5)}.$$

Here, a small value of $\tilde{\alpha}_s$ corresponds to a small supply (i.e., an adverse supply shock). Thus, the speculator only manipulates at expiration when supply is low. Moreover, the likelihood of manipulation is larger, the larger is x. Furthermore, note that the critical value of the supply shock is linear in x. All of these features were found in the model in the main text.

When a manipulation occurs, the manipulator sells futures contracts at the price:

$$P_1 = -\frac{\tilde{\alpha}_s}{\beta_s} + \frac{1}{\beta_s} \frac{\alpha_d \beta_s + \tilde{\alpha}_s \beta_d + x(\beta_d + M\beta_s)}{2[\beta_d + (M + 1)\beta_s]}.$$

Note that this is a linear function of x. Thus, as in the model in the main text, the expected price at delivery in market 1 is a quadratic function of x, the variance of this price is a cubic function of x, as is the covariance between this price and the price in the other markets (and the consumption price). Finally, due to these facts, the first order condition is a quartic function of x. Therefore, the main features of this model are identical to those of the model in the main text.

Note further that two main results–that x is increasing in Y and decreasing in T_T–hold in this model as well, as the first order condition is the same in the two models. Moreover, the result that $x \leq .5MY$ also holds because it is possible to prove (using an analysis very similar to that presented in the main text) that:

$$\frac{\partial \sigma_1^2}{\partial x} \geq 2 \frac{\partial \sigma_{1c}}{\partial x}$$

This, along with the facts that $\sigma_{1c} \leq \sigma_1^2$ and $\partial \bar{P}_1 / \partial x > 0$ implies that $dW/dx < 0$ when evaluated at $x = .5MY$. Thus, as in the main model, the large speculator chooses $x < .5MY$. Therefore, all of the analytical results derived in the main text hold in this alternative model.

Moreover, numerical solution of this model generates all of the results for the welfare effects of manipulation, its effect on the risk premium, and the effect of position limits, as presented for the main model. There is one difference between models that deserves some comment. Whereas in the main model, manipulation has no effect on prices in markets $2, \ldots M$, in this model a manipulation depresses prices in these other markets. In essence, manipulation stimulates excessive production in market 1 to satisfy the manipulator's inefficiently large demand for deliveries. His dumping of this excessive delivery market output post-delivery depresses the prices in the other markets. That is, the burying the corpse effect is spread among all the various markets. This encourages manipulation, because the manipulator incurs a lower cost from disposing of units delivered to him than is the case when the burying the corpse effect falls exclusively on the price in the delivery market (as in the main model). Further, burying the corpse is less costly for a given quantity of deliveries, the flatter the supply curves in the various out-of-position market, because in this case output adjustment in the out-of-position markets cushions the price impact of the market 1 output distortion.

This analysis is sufficient to show that the main implications of the model presented in the text continue to hold even under very different assumptions about the structure of the market. In particular, the model in the appendix permits some inter-connections between markets because the demand in any one market is a derived demand that depends on supply in the other markets. This is a plausible condition for many important commodities.

It is possible to extend the model even further to permit the possibility of transportation between production regions. Specifically, consider the case where the cost of shipping between any production market $2, \ldots, M+1$, and market 1 is τ per unit of the commodity. In this case, whenever the difference between P_1 and the prices in the other markets exceeds τ, total supply of the commodity at this price in markets $2, \ldots, M+1$ is available for delivery. This occurs when the large speculator takes a sufficiently large volume of deliveries, because these deliveries (a) drive up the price in market 1, and (b) depress the prices in the other markets. Thus, at this critical level of Q (denoted by $\hat{Q}$), which depends on the realization of $\tilde{\alpha}_s$, there is a discontinuity in the supply of deliveries. Unless x is extremely large, the large speculator will never take more than $\hat{Q}$ deliveries. In this model, the relation between P_1 and x will be piecewise linear in $\tilde{\alpha}_s$, with three different segments. The first segment corresponds to large supply, when manipulation does not occur. The second segment is for a sufficiently adverse supply shock, in which event the manipulator takes deliveries from only market 1, and the difference between the price in market 1 and the price in the other markets is less than τ. The third segment is for very small values of the supply shock, in which case the speculator takes the number of deliveries that makes the inter-market price differential to equal τ exactly. The location of the break points depends on x.

Since P_1 is a linear function of x in each of these segments (with a slope of zero in the first and third segments, and a positive slope in the second), all of the results derived in the main model, and the model analyzed formally in the appendix follow. Indeed, it is possible to allow the transport costs to market 1 to differ between markets $2, \ldots, M+1$. For each different transportation cost value, there is a discontinuity in the marginal cost of deliveries function, the location of which depends on x and $\tilde{\alpha}_s$. The analysis of Pirrong (1993) implies that in this case, the large speculator will choose to take deliveries only from market 1, or in an amount corresponding to one of the discontinuities. If, for instance, the speculator takes deliveries in an amount where the fourth discontinuity in the supply of deliveries curve occurs, he will take deliveries from market 1, and the markets with two lowest transportation costs. The linearity assumptions for supply and demand ensure that the analytical results continue to hold even in this more complicated model.

Thus, the results obtained for the model presented in the main text continue to hold in the presence of connections between the physical markets, either indirectly *via* derived demand linkages arising from supplying a common consumption market, or directly *via* transportation.

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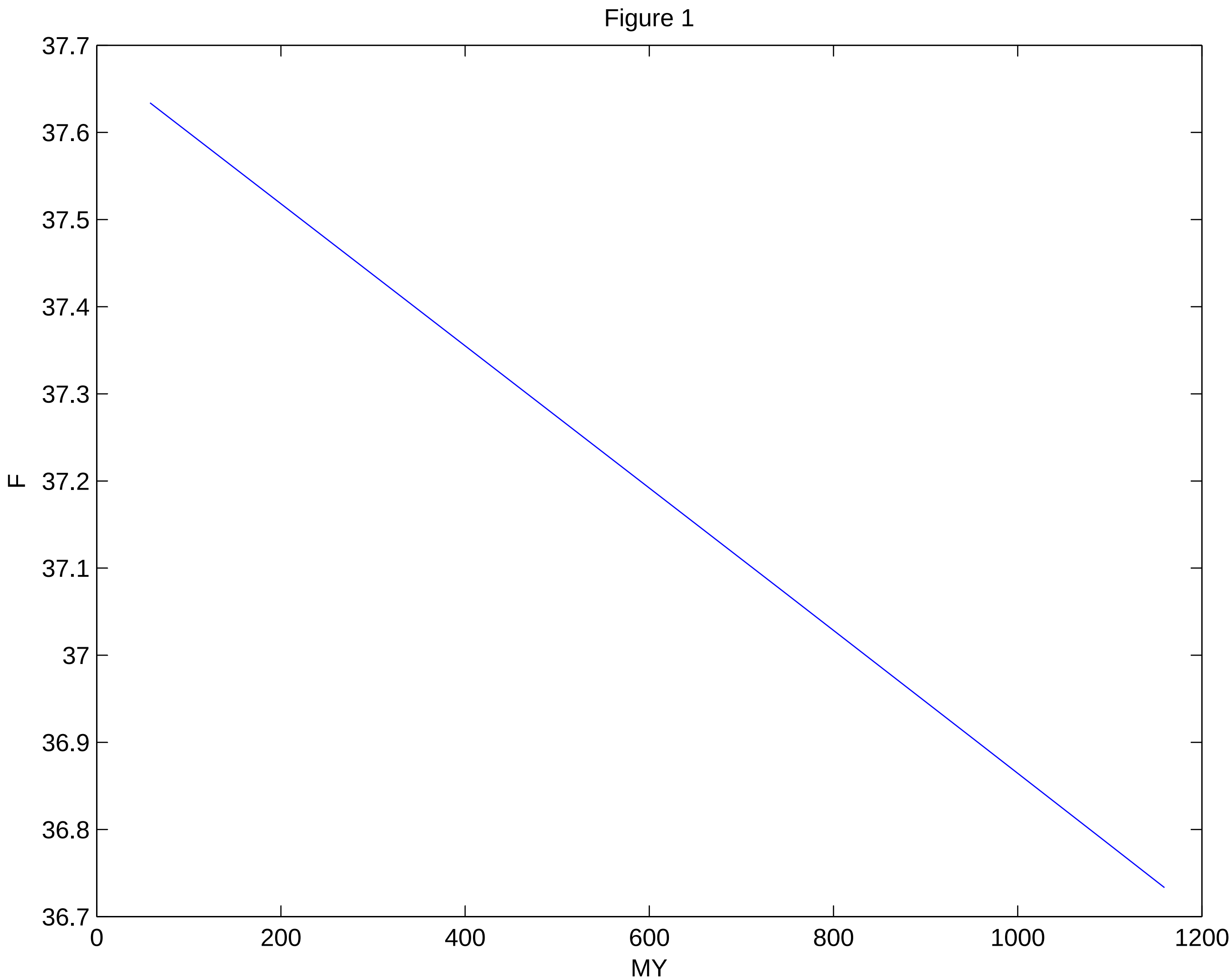

Figure 1
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Figure 2

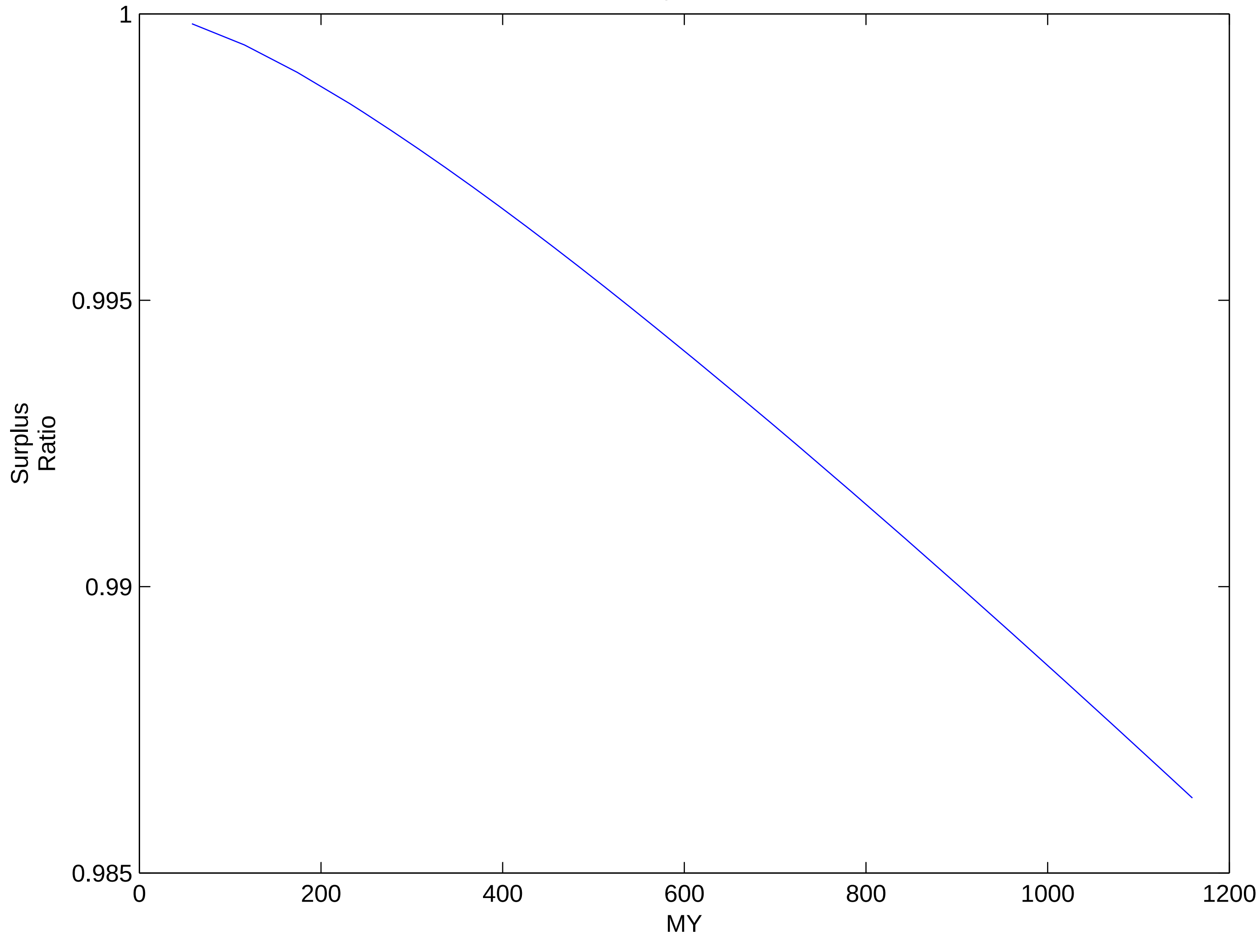
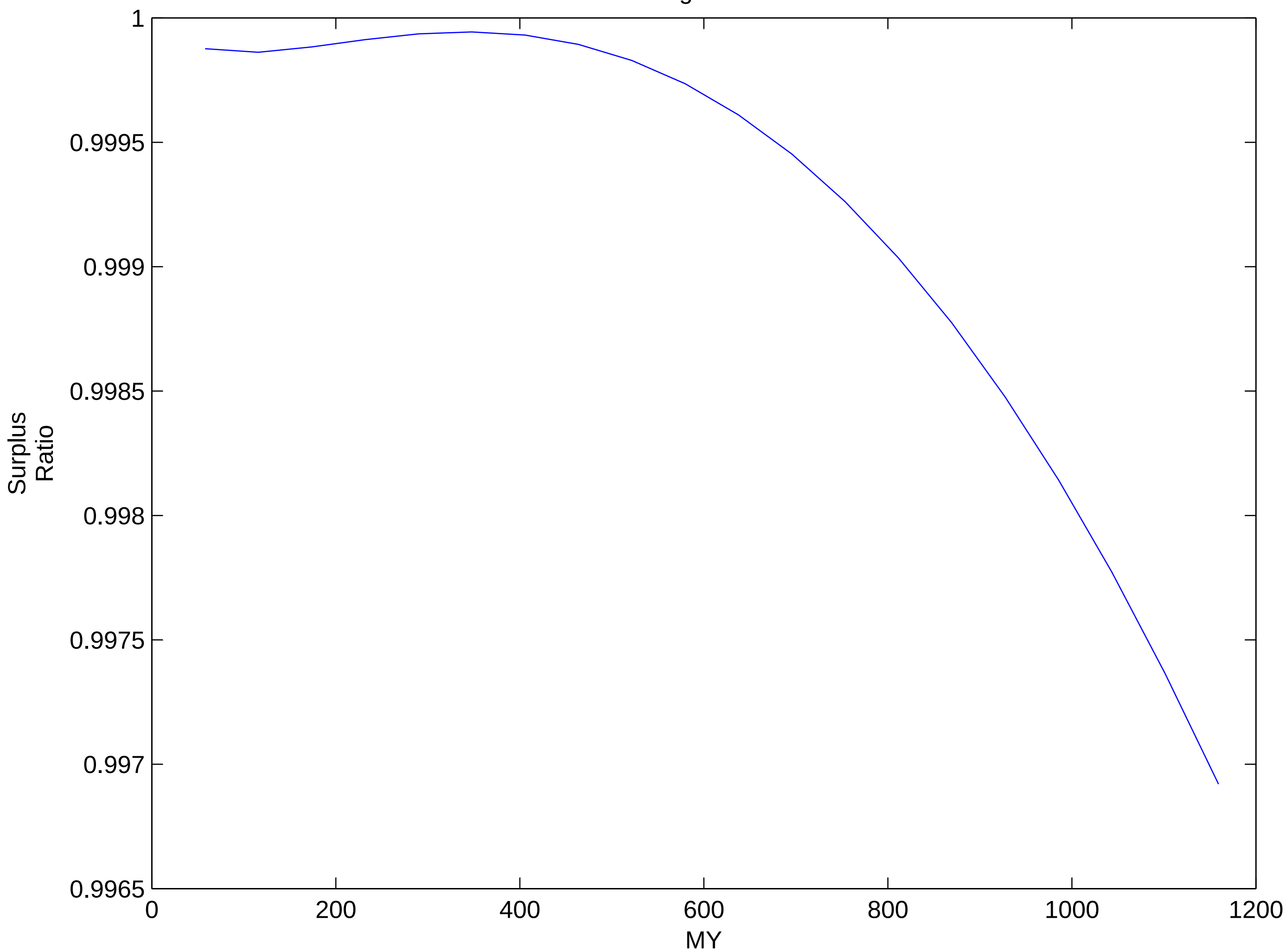


Figure 3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