

Squeeze Play: The Dynamics of the Manipulation End Game

Craig Pirrong
Bauer College of Business, University of Houston
Houston, TX 77204
713-743-4466
cpirrong@uh.edu

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1 Introduction

Market power manipulation—commonly known as a “corner” or “squeeze”—is one of the main regulatory and legal challenges facing derivatives markets. There are many famous examples of squeezes dating back to the very origins of derivatives trading, and extending to the present day.¹

These manipulations are inefficient. The exercise of market power results in distortions in the production, consumption, and transportation of the commodity underlying the derivatives contract. Moreover, manipulation

¹The soybean market was rocked by a major manipulation in 1989, copper was massively manipulated in the mid-1990s, Brent crude squeezed at various times in the 1990s and 2000, and recently there have been allegations of corners in aluminum and propane. Nor are corners limited to physical commodity markets. There is evidence of squeezes in US Treasury Bond futures in 1986 (Cornell and Shapiro, 1989); Salomon Brothers squeezed the Two-year Treasury Note market in 1991 (Jegadeesh, 1993); the UK Long Gilt contract was squeezed in the 1990s (Merrick, Naik, and Yadev, 2005); there are allegations that a corner of the Ten-Year Treasury Note futures contract (one of the world’s largest) occurred in 2005; and the US Treasury and Federal Reserve have expressed serious concerns about chronic squeezes in the Treasury repo and futures markets (Clouse, 2006). There are also indications that credit derivatives have been squeezed in the aftermath of credit events. Allen, Litov, and Mei (2006) document a number of stock corners.

causes prices to move unpredictably in response to factors other than supply and demand fundamentals; that is, manipulation injects noise into derivatives prices that reduces price informativeness and thereby undermines the utility of derivatives markets as loci of price discovery. Furthermore, market power manipulation injects noise into price relations (e.g., basis relations) that reduces the hedging effectiveness of derivatives contracts. For all these reasons, manipulation is proscribed by statute in the United States, and reducing the frequency of manipulation is the primary focus of regulatory efforts in derivatives markets.

Despite the importance of manipulation to the efficiency of derivatives markets, many aspects of the economics of corners and squeezes are not well understood. In particular, the dynamics of trading as a contract nears expiration have not been modeled extensively, and as a result the existing literature cannot capture many of the interesting actions and interactions observed during actual squeezes. This article fills that void by examining the effects of asymmetric information on the trading strategies of large longs and shorts as a contract approaches expiration.

A traditional market power manipulation has a well defined end game: the holder of a dominant long futures (or forward) position can demand excessive deliveries—or equivalently, agree to sell contracts only at a super-competitive price. Pirrong (1993) presents a formal model of this end game based on implicit assumptions regarding the bargaining process that the dominant long employs; specifically, Pirrong assumes that there is no private information, and the large long makes a single price take-it-or-leave offer. Shorts then decide how many contracts to liquidate by buying from the long at that price, and how many to close by making delivery.

Although the end game model accurately describes many phenomenon

observed in historical corners, and makes predictions regarding the factors that make a market susceptible to the exercise of market power by a large long, the bare-bones nature of the model means that it cannot capture some of the rich and varied phenomena observed in real-world corners. Specifically, it cannot capture some of the complex interplay between longs and shorts in the period leading up to the end of trading of a cornered contract; the fact that some large longs liquidate all or part of their positions prior to the very end of trading; the spectacular failures of corners, where the large long was inundated with unexpectedly large deliveries, leaving him with a highly unprofitable “corpse to bury”; and the long’s use of “step up” orders to liquidate positions.

The end game in the Pirrong model also poses a puzzle. In that model, there are substantial deadweight costs and transfers of wealth from the shorts to parties other than the large long—specifically, to small, non-manipulating longs and the owners of the commodity that the shorts buy in order to deliver. The deadweight cost and wealth transfers create potential gains from trade that the large long and the shorts can split. This raises the question of whether a richer bargaining and trading mechanism that permits opportunities for traders to make mutually beneficial transactions leads to different conclusions regarding the playing of the end game and the welfare effects of manipulation.

In this article, I address these issues by incorporating incomplete information and multiple trading rounds into the end game model. The first set of models assumes that the large long’s position is private information. Shorts and small longs may obtain a signal on this position, but they cannot observe it without error. This private information impedes negotiations between the long and the shorts; due to the private information, some gains

from trade are not realized. Nonetheless, some gains are captured through pre-end game liquidations.

Two main results obtain when the manipulator's position is private information. First, if all shorts are atomistic, no trade occurs before the expiration of the manipulated contract, and the only equilibrium is the inefficient end game outcome. This occurs because (a) the atomistic shorts face a coordination problem, and (b) there are externalities among the shorts. If some shorts liquidate prior to expiration, the long's market power is reduced, and the shorts who do not liquidate prior to expiration are able to do so at a lower price at expiration. Under these circumstances, the dominant strategy for each small short is to abstain from trade (or equivalently, bid the competitive price) prior to expiration. The end game therefore occurs at expiration, with all its associated inefficiencies.

Second, if there is a large short, some of the externality is internalized, and the coordination problems are mitigated. Under these conditions, the large short may find it profitable to trade with the large long prior to expiration because he can internalize some of the gains from trade with the long. Even here, however, information asymmetry and coordination problems preclude the realization of all gains from trade. In order to extract rents from middle-sized longs, when the long's position is private information, the large short does not bid aggressively enough to induce bigger longs to trade prior to expiration even though the gains from trade are largest with the largest longs.

This model makes several interesting predictions. Some trade occurs prior to expiration, and the delivery end game may not occur at all. If the large long trades with the large short prior to expiration, some of his position is liquidated at a supercompetitive price, and the price falls after

this trade; it may rise again at expiration, or continue to fall. If the large long does not trade with the large short (and the larger longs will not trade when information about position is private), the price rises subsequent to the rejection of the large short's offer, and then may fall or continue to rise at expiration. Thus, the model predicts price movements around expiration not driven by fundamentals, and which may be associated with pre-expiration trades. Pre-expiration trade improves welfare (by reducing deadweight losses) and transfers wealth from small longs (who otherwise free ride on the manipulative long) to other traders.

The second model explores the implications of private information about the cost of delivery. In this model, the shorts have private information about the marginal cost of delivery that the large long does not possess. When deciding the price at which he is willing to liquidate, the large long must optimize over the distribution of possible delivery marginal costs. Sometimes the large long overestimates the cost of delivery, and in the event, demands too high a price to liquidate. In this case, shorts deliver large quantities, potentially imposing a substantial loss on the large long.

The last model provides a rationale for the use of step up orders. If shorts have different costs of delivery, and private information about these costs, during the end game a large long may increase profit by using non-linear pricing. Presenting shorts with a menu of price-quantity combinations screens shorts according to their delivery costs. This menu can be implemented through step up orders. The resulting equilibrium requires that the menu exhibit monotonicity properties. I evaluate the actual step up orders used in some alleged manipulations to determine whether these properties hold.

In brief, the incorporation of private information allows for richer interac-

tions and behavior in a market susceptible to manipulation. The predictions of these models help explain events that have occurred in real world corners and squeezes, including some historically important ones. The models also help to explain the “technical” fluctuations in prices that can occur around the expiration of a futures contract, i.e., price fluctuations not driven by supply and demand fundamentals.

This paper is closest in spirit to that of Donaldson and Cooper (1997). They analyze the strategic interactions between longs and shorts as a futures contract approaches expiration where one large trader may have built up a cornering interest. Donaldson-Cooper assume all information about positions is public, however, and their results are driven primarily by the externalities among shorts and longs and their influence on bidding behavior, and the assumptions about the trading process, most notably the assumption that only one unit is transacted in each round of trade. As is the case with the present model, Donaldson and Cooper predict that technical factors can lead to price fluctuations (including bubble-like behavior) as a futures contract approaches expiration. Their model does not predict partial liquidations by the large long, after which the large long still exercises market power during the end game. Their model also does not predict that the existence of large shorts can exert a decisive impact on the pre-expiration trading process.

The remainder of this article is organized as follows. Section 2 reviews the delivery end game and discusses how the inefficiencies and wealth transfers that occur during the end game create potential gains from trade prior to expiration. Section 3 shows that despite the existence of these gains from trade, there is no pre-expiration trading when all shorts are atomistic; in this case, the only equilibrium is that the long liquidates his entire position

in the end game. Section 4 adds a large short to the analysis, and demonstrates that in the presence of private information the large short makes an offer to trade that some large longs accept. The short's price is above the competitive price, and trades off rent extraction against efficiency gains of avoiding the end game. Section 5 shows how private information about delivery costs can result in the large long rationally choosing a price at expiration that results in an avalanche of deliveries that makes the squeeze highly unprofitable *ex post*. Section 6 extends the analysis to show how private information about delivery costs can induce the large long to use step up orders at expiration. Section 7 summarizes.

2 The Delivery End Game

Consider a futures contract traded on the commodity. The contract is settled by delivery in market 1.² Demand in this market is $P = \theta_D - \phi_D q$, where θ_D and $\phi_D > 0$ are constants, P is the spot price in the market, and q is the quantity consumed. Supply in the market is $P = \theta_S + \phi_S q$, where $\phi_S > 0$ is a constant and q is the quantity supplied to the market.³ Initially, I assume that all supply and demand parameters are common knowledge; I allow private information on these parameters in section 5. If the delivery market is competitive, the competitive quantity in the market is determined by the intersection of the supply and demand curves, and equals:

²Pirrong (2001) shows that the same results would obtain in a market where the futures are cash settled against the spot price in market 1.

³The delivery demand and supply conditions differ from those in Donaldson-Cooper. They assume a stair-step delivery cost function. This discontinuity leads to different end game pricing and efficiency implications from those derived when delivery costs are continuous, as is the case here.

$$Q_c = (\theta_D - \theta_S)/(\phi_D + \phi_S)$$

There is a large long in the market, and a competitive fringe of atomistic longs.⁴ The large long has a position of x contracts, and the price taking atomistic longs have an aggregate position of $I - x$, where I is the total open interest in the contract. At expiration, the large long makes a take-it-or-leave-it offer of P_m to the outstanding shorts; this is the price at which he is willing to sell contracts to the shorts, thereby liquidating their positions. Shorts then decide whether to accept this offer and liquidate by buying futures, or instead to satisfy their contractual obligations by delivering the commodity.

If the large long chooses a price P_m , shorts choose to delivery a quantity Q such that the marginal cost of delivery equals P_m . That is, to minimize their costs of closing their positions, shorts choose to deliver to the point that $\theta_S + \phi_S Q = P_m$. Since there is a one-to-one relationship between the price that the large long chooses, and the quantity of deliveries, the large long's price decision is equivalent to choosing the number of deliveries to maximize his delivery period-revenues. For a given choice of Q , the long sells futures $x - Q$ futures contracts at a price equal to the marginal cost of delivery of Q units.

$\Pi(x)$ is the large long's revenue at futures contract expiration. This revenue consists of two components: the revenue from sales of the commodity

⁴The trading process by which traders accumulate positions is not modeled here. Pirrong (1994) shows that large traders can utilize randomized trading strategies to accumulate positions sufficiently large to permit them to squeeze the market during the end game. The prices at which the long can acquire positions depends on the nature of the end game. As a consequence, the introduction of pre-end game trading affects trading at the position-accumulation stage.

that is delivered to him, and revenue from the sales of futures contracts.

Thus,

$$\Pi(x) = \max_Q \{Q(\theta_D - \phi_D Q) + (x - Q)(\theta_S + \phi_S Q)\} \quad (1)$$

In this expression, Q is the number of deliveries that the large long takes.

Due to the existence of an upward sloping supply curve to the delivery market, a long with a sufficiently large position can squeeze the futures contract. To squeeze, the speculator demands deliveries that exceed the competitive quantity in the delivery market (Pirrong, 1993). Therefore, the price in the delivery market at expiration (and hence the futures price at expiration) is higher than the competitive price if and only if $Q \geq Q_c$.

By demanding excessive deliveries, the manipulator forces excessive production of the commodity, driving up the marginal cost of production; shorts must pay this inflated marginal cost of production in order to acquire the good for delivery, and hence they are willing to pay this inflated price to purchase contracts to extinguish their obligation to make delivery. Immediately following expiration, the price in the delivery market falls below the competitive equilibrium price because the manipulator dumps the excessive supplies of the commodity in market 1. This post-delivery fall in price is referred to as the effect of “burying the corpse,” the corpse being the large quantities of deliveries the long must take to inflate prices. The cost of burying the corpse (i.e., disposing the excessive deliveries at a depressed price) affects the profitability of manipulation, and the squeezer takes this effect into account when deciding how many contracts to liquidate and how many to close via delivery. This cost also affects the large long’s incentive to bargain prior to the end game.

Solution of the first order conditions for (1) implies:

$$Q = \frac{\theta_D - \theta_S + \phi_S x}{2(\phi_D + \phi_S)}$$

$Q \geq Q_c$ when:

$$x \geq \hat{x} = \frac{\theta_D - \theta_S}{\phi_s}$$

That is, \hat{x} is the smallest long position such that a squeeze occurs. If $x < \hat{x}$, the large long does not squeeze, and the contract liquidates at the competitive price $P_c = \theta_S + \phi_S Q_c$.

It is readily shown that if $x \geq \hat{x}$:

$$P_m(x) = \theta_S(1 - .5A) + .5\theta_D A + .5\phi_S A x$$

where $A = \phi_S / (\phi_D + \phi_S)$. This can be rewritten as:

$$P_m(x) = \mathcal{A} + \beta x.$$

Recall that open interest is I contracts, meaning that there are I short positions outstanding. Shorts pay either $P_m(Q)$ to obtain the deliverable commodity, or the same price to buy back their futures positions. Thus, shorts in aggregate incur a cost of $P_m(Q)I$ at delivery. Absent a squeeze, they would pay $P_c I$. Thus, the total cost of a squeeze to the shorts is $I(P_m(Q) - P_c)$.

The large long pockets $\Pi(x)$. Note that because $\theta_D - \phi_D Q < P_c < P_m(Q)$ when $Q > Q_c$ (due to the necessity of “burying the corpse”), $\Pi(x) < xP_m(Q)$. Moreover, since $x \leq I$,

$$\Pi(x) - P_c x < I(P_m(Q) - P_c)$$

That is, the squeezer’s profit (squeeze revenue minus the revenue he would have earned by liquidating his entire position at the competitive price) is smaller than the cost that the squeeze imposes on the shorts.

The difference between the shorts' losses and the squeezer's gain can be broken into four parts:

- Deadweight loss. By demanding excessive deliveries, the large long induces a distortion in flows of the commodity to the delivery market, and consumption in that market. The marginal cost of the additional units delivered exceeds their marginal value to consumers.
- Transfers to the atomistic longs. Atomistic longs can liquidate their positions at the manipulated price (or slightly below, to ensure that they take no deliveries.) Thus, the atomistic longs liquidate their positions at a supercompetitive price, and thereby earn a rent at the shorts' expense.
- Transfers to owners of the commodity. Owners (or producers) of the commodity can sell it to shorts at the manipulated price. Since the marginal cost curve slopes up, the commodity owners/producers earn a surplus on the units of the commodity delivered. This surplus equals the windfall gain realized by selling the competitive quantity at a supercompetitive price, and difference between the revenue realized by selling more than the competitive quantity at the supercompetitive price, minus the cost of producing these additional units.
- Transfers to consumers of the commodity. The post-delivery price for the commodity is less than the competitive price due to the attraction of excessive supplies to the delivery market. Consumers of the commodity reap the benefit of the lower price, but this lower price imposes a cost on the manipulating long.

Due to the difference between the shorts' losses and the manipulator's

gain, these parties can realize gains by negotiating a settlement that eliminates the need to play the end game. The end game outcomes define the bargaining range, but there are transactions that would make both the shorts and the large long better off. Some of these transactions would transfer wealth from the atomistic longs and the owners and consumers of the commodity, but they also have the potential to improve welfare by reducing the distortions in production and consumption that occur if the end game is played.

This raises the questions: Will the large long and the shorts come to a mutually beneficial bargain? If not, why not?

The following sections explore these questions, and show that the answer depends on the structure of the short side of the market, and the existence of private information. In the presence of private information, when shorts are small, no bargain is possible due to externalities and coordination failures; thus, with atomistic shorts, the end game is always played if $x > \hat{x}$. If there is a large short, however, under some circumstances a mutually beneficial trade occurs prior to the end game.

3 Pre-Expiration Trading With Atomistic Shorts and Private Information on the Large Long's Position

It is well known that private information can prevent the negotiation of mutually beneficial transactions. In this section, I assume that the large long knows his position with certainty, but that shorts only know the distribution of his position.

This assumption is realistic. A trader certainly knows his own position. Moreover, traders are not obligated to disclose their positions to the mar-

ketplace at large; indeed, they often take extraordinary efforts to conceal their positions from others.⁵ For instance, one reason traders use brokers is to conceal their identities from counterparties. Although exchanges and regulators collect position information, they are typically precluded from disclosing this information to other market participants. Moreover, by trading with multiple counterparties, an individual trader prevents any individual counterparty from knowing his entire position.

That said, given salience of position information, market participants invest real resources in an attempt to learn about the positions accumulated by other traders. Most large trading firms have commercial intelligence networks. Brokers provide information on trading activity, including information relating to what types of firms are trading and in what volumes. In the Treasury market, for instance, this is referred to as “market color” and represents a major service that brokers provide their customers.

Although market participants value this service (as indicated by their willingness to pay for it), it provides only a noisy signal on the position held by any one trader because (a) an individual broker does not see all market activity, (b) brokers’ fiduciary obligations preclude them from disclosing the names of actual transactors, and (c) traders take active measures to conceal their activities from brokers’ prying eyes.

In some markets, the government produces data that provides some information on positions. For instance, in the United States, the Commodity Futures Trading Commission provides Commitment of Trader Reports. These reports disclose the aggregate position held by the four and eight

⁵Easterbrook (1986) emphasizes the role of secrecy in manipulations. Indeed, he identifies manipulation as a species of fraud because the manipulator relies on concealment of his position and intentions from his counterparties in order to lure them unsuspecting into a corner.

largest longs, and the four and eight largest shorts. However, these reports (a) reflect positions held some time prior to their release, (b) aggregate positions across at least four longs, and (c) aggregate positions across all delivery months for a particular commodity. Therefore, they provide an imprecise measure of the current largest long position in a particular contract.

Thus, market participants have some information regarding the positions held by others in the marketplace, but they cannot know any individual trader's position with certainty. To formalize this reality, I assume that all traders but the large long receive a signal on the large long's position. To simplify matters, I assume that all traders receive the same signal, and the large long knows the signal that they receive. Moreover, all traders agree that the signal implies that the distribution of the large long's position is given by the continuous density function $f(x)$, where x has support $[x_o, I]$.

In this section, I assume that all shorts are atomistic price takers. That is, the shorts live on a continuum of length 1. These shorts, the atomistic longs, and the large long have an opportunity to trade prior to the expiration end game. The end game occurs at t_2 , and at $t_1 < t_2$, shorts can submit bids indicating the price at which they are willing to purchase contracts. The atomistic longs can also submit offers to sell. I assume that the atomistic traders' limit orders are crossed, and that the large long can trade by market order with the remaining, uncrossed bids. All positions remaining open at t_1 are closed in the delivery end game at t_2 .

Assume initially that all shorts adopt a symmetric trading strategy, bidding $b(\tilde{x}) > P_c$ after receiving signal \tilde{x} on the large long's position. It is straightforward to see that this cannot be an equilibrium strategy. The argument proceeds in several steps. First, given this bidding strategy, the large long either hits all or none of the bids. This is true because the marginal rev-

enue of hitting an additional bid is $b(\tilde{x})$, and his marginal cost after hitting q bids is:

$$-\frac{d\Pi(x - q)}{dq} = P_m(x - q)$$

where this result obtains by applying the envelope condition to (1). Since $P_m(x - q)$ is decreasing in q (the end game price is lower, the lower is the large long's position), if $P_m(0) = P_c > b(\tilde{x})$, the long trades zero contracts at t_1 . If $P_m(0) < b(\tilde{x})$, since marginal cost is decreasing the long's optimum is a corner solution; he either sells x contracts or none. That is, the marginal cost curve crosses the marginal revenue curve from above, meaning that the intersection is a local minimum, and the maximum is a corner solution.⁶

Consider an individual short. All other shorts choose $b(\tilde{x})$; does this short have a better strategy? Yes: he can bid $P_c - \epsilon$. The large long never hits this bid.

If the long hits the other shorts' bids, he sells so many contracts that his position falls below \hat{x} , the minimum required for a corner, and during the end game the price equals P_c . The defecting short can then liquidate his position at this price. Conversely, if the large long doesn't hit the bid, the short liquidates at $P_m(x) > P_c$. If this short also bids $b(\tilde{x})$, the short pays $b(\tilde{x})$ when the large long hits the bids. If the large long does not hit the bids, the short must cover during the end game at a price $P_m(x) > P_c$. Thus, bidding P_c dominates bidding $b(\tilde{x}) > P_c$. This produces:

Result 1. There is no symmetric equilibrium in which atomistic shorts all bid at a single price in excess of the competitive price.

⁶In fact, the large long is willing to hit all I bids as the shorts' bids exceed the competitive price that prevails at expiration once the large long liquidates. The large trader can therefore sell I contracts at $b(\tilde{x})$, and cover the resulting short position of $I - x$ contracts at a price of P_c .

Moreover, it is an equilibrium for all shorts to bid P_c . Assume that all atomistic shorts but one follow this strategy. Should this one short bid $\hat{b} > P_c$? No: the large long hits the bid if $\hat{b} > P_m(x)$ because $P_m(x)$ is the marginal cost of hitting this bid. In the event, this short pays $\hat{b} > P_m(x)$ to buy back his short position. Conversely, if he bids P_c , the long does not hit any offers, the end game is played, and the short covers his position at a price $P_m(x)$. Thus:

Result 2. It is a Nash equilibrium for all atomistic shorts to bid P_c .

Indeed, this is the only equilibrium. Assume that short $i \in [0, 1]$ bids $b_i(\tilde{x}) > P_c$. Due to price priority, these bids are ordered from lowest to highest, and the market clearing price is determined by the bid of the marginal short. Due to the ordering of bids, the large long faces a downward sloping demand curve for his position at t_1 , and hence the large long's marginal revenue curve is below the demand curve. If the large long maximizes his profit by choosing to liquidate $q \leq x$ contracts, at this point the long's marginal revenue equals his marginal cost, which is $P_m(x - q)$. All shorts whose bids are hit pay a price that exceeds this marginal cost. All shorts whose bids are not hit pay a price of $P_m(x - q)$ in the delivery end game. Therefore, not trading at t_1 strictly dominates trading at t_1 . As a result, a short is better off by cutting his bid to P_c . Thus:

Result 3. The unique equilibrium is for all shorts to bid P_c . As a result, there is no trade at t_1 , and the delivery end game is played at t_2 , with all shorts paying $P_m(x)$ to cover their positions.

Therefore, despite the gains from trade between the shorts and the long, no pre-expiration trade occurs. This is because atomistic shorts face an extreme winner's curse. The curse arises from an externality across shorts. Any short that buys at t_1 reduces the large long's market power; this benefits

all other shorts who do not liquidate. So each short would prefer to free ride on the other shorts. As a consequence, no short bids aggressively, and none of the potential gains of trade between the shorts and the large long are realized.

4 Pre-Expiration Trading With A Large Short and Private Information on Positions

Atomistic shorts face severe free rider and coordination problems. This suggests that the stark results of the previous section may be relaxed if there is a large short who internalizes some of the externality. This section demonstrates that this is indeed the case, but that the problem is not eliminated if there is private information.

This section explores a modified and extended model. In this model, there is a large short with a position of $S \leq I$. For simplicity, I assume that S is public information.⁷ The large short and a fringe of atomistic shorts receive a signal on x . Subsequent to the receipt of the signal, at t_0 , the large short can negotiate with the large long. During the negotiation the large short bids a price P per unit for up to S contracts. The atomistic longs are excluded from this negotiation. The results of this negotiation—the price and the amount of contracts the long sells—are disclosed to the market. At t_1 , the small shorts and small longs can trade. At t_2 , remaining positions are closed in the delivery end game.

The assumption that the atomistic longs cannot hit the large short's bid can be justified when there is the possibility of private settlements between the short and the long. There are numerous examples of such private settle-

⁷This assumption is innocuous here as the short has all the bargaining power.

ments during corners on the Chicago Board of Trade, as described in Taylor (1917). Similarly, private negotiations are feasible in over-the-counter markets.

If all bids must be made publicly and accessible to all subject to priority and precedence rules, however, atomistic longs may hit bids, and be allocated trades. However, I derive below a sufficient condition (that depends on S and $f(x)$) that ensures that atomistic longs do not hit the large short's bid even if they have the opportunity to do so. I also provide examples in which this sufficient condition holds.

Moreover, it should be noted that atomistic longs face an adverse selection problem that strongly limits their incentive to hit the short's bid. I show below that there exists a critical position size x^* such that a long with $x \leq x^*$ hits the short's bid, but that one with $x > x^*$ does not. Regardless of the priority and precedence rules (e.g., time priority, size precedence, or pro rata allocation), an atomistic long is more likely to have his order filled when $x > x^*$ than when the opposite is true, as he faces no competition from the large long in this case. But, this is exactly when the atomistic long would prefer *not* to be filled, as this is when the price during the end game is high. The atomistic long prefers to be filled when $x \leq x^*$, as if he is not (or if he does not bid), in this case the price falls below the short's bid in the next round of trading (as I also show below). But, since the large long hits the short's bid when $x \leq x^*$, the atomistic long faces more competition to execute at P , and is less likely to get a fill at this price. Under size priority, he will not be filled; under pro rata rationing, his probability of being filled is essentially zero. Only under strict time priority is the probability of being filled appreciably bigger than zero, but even under this rule it is less than one. Thus, atomistic longs may not hit the short's bid even when

the sufficient condition does not hold (as this condition ensures that a small long does not hit the bid even if his order has absolute priority and hence he faces no adverse selection-driven rationing.)

The assumption that the large short bids a common price for all S units is made for the purpose of tractability, but in general, the large short does not submit a downward sloping bid schedule with no constraint on the quantity that the large long sells; the long picks off the short's high bids, and then closes the remainder of his position at the squeeze price in the delivery end game. Optimization requires the short equate the marginal cost of repurchasing a contract at t_0 and the marginal cost of repurchasing a contract in the end game, but this cannot occur if the short submits a downward sloping bid curve. As noted in section 3, the large long equates the marginal revenue of liquidating at t_0 to the marginal value of a contract at t_2 . Since the t_2 marginal value is the end game price, the marginal revenue at t_0 is also the end game price. If the short submits a downward sloping bid schedule, this t_2 price must be less than the marginal bid that the long selects at t_0 (because with a downward sloping schedule the marginal bid is above the marginal revenue).⁸ Thus, if the short submits bids at different prices, the marginal price he pays at t_0 exceeds the marginal price paid at t_2 . As a result, submitting bids at different prices cannot minimize the cost of liquidating his position.

It can be shown, using the same arguments as in the previous section, that small shorts desire to free ride, and bid P_c . Thus, the analysis can be

⁸However, the short may want to use a menu of "all-or-none" bids at different prices and quantities which require the long to trade the entire quantity at the price selected. This allows the short to discriminate more effectively among longs with different values of x . The appendix presents the analysis when such orders are allowed. Most of the qualitative implications of the analysis are the same as those derived in the main text.

restricted to the choice of the large short.

If the large short chooses a price P and bids for S contracts at this price, some large longs accept and some reject. The long's decision depends on x . A large long that hits the bid earns a revenue of:

$$\Pi_H(x) = SP + \Pi(x - S)$$

whereas a long that does not hit the bid receives $\Pi(x)$. The long hits the bid if and only if $\Pi_H(x) > \Pi(x)$, that is, if $\Pi(x) - \Pi(x - S) < SP$.

For a given P , there exists an x^* such that:

$$\Pi(x^*) - \Pi(x^* - S) = SP$$

Since the left hand side of this expression is increasing in x , any long with $x > x^*$ will not hit the short's bid, but any long with $x \leq x^*$ will do so.

Note that $\Pi(x) = \Pi(x - s) + \int_0^S P_m(x - y)dy$. Therefore, since $P_m(u) = \mathcal{A} + \beta u$:

$$SP = \mathcal{A}S + \beta x^*S - .5\beta S^2$$

so:

$$x^*(P) = \frac{P - \mathcal{A} + .5\beta S}{\beta}$$

Moreover,

$$\frac{dx^*}{dP} = \frac{1}{\beta}$$

The large short chooses P to minimize his expected cost of closing his position. He pays PS if $x \leq x^*$, but must pay $P_m(x)S$ for $x > x^*$. Therefore, the short's expected cost is:

$$C(P, S) = PSF(x^*(P)) + S \int_{x^*(P)}^I P_m(x)f(x)dx$$

where $F(\cdot)$ is the cdf of the distribution of x conditional on the signal the short receives. The first order condition for the minimum implies:

$$P = P_m(x^*(P)) - \frac{\beta F(x^*(P))}{f(x^*(P))} \quad (2)$$

Recalling that $P_m(x) = \mathcal{A} + \beta x$, and that $x^* = (P - \mathcal{A})/\beta + .5S$

$$P = \mathcal{A} + \beta \left[\frac{P - \mathcal{A}}{\beta} + \frac{1}{2}S \right] - \frac{\beta F(x^*(P))}{f(x^*(P))}$$

Simplifying produces:

$$\frac{1}{2}S = \frac{F(x^*)}{f(x^*)} \quad (3)$$

This can be solved for x^* , which in turn implies the large short's choice of P . Note that since $F(x_0) = 0$, if $F(x)/f(x)$ is increasing in x (as is the case for most single-peaked parametric densities), $x \in (x_0, I]$.

According to (2), the large short offers a price that is below the price at which some longs would liquidate their position in the delivery end game. These longs are willing to liquidate at this lower price nonetheless because they realize only a fraction of their end game price as a profit (due to dead-weight losses and the cost of burying the corpse.) Thus, the liquidating long and large short split some of the gains from trade described above. The large short must trade-off the benefit of buying some contracts at a price lower than he would pay during the delivery end game against the cost of buying some contracts at a price higher than he would pay during the end game. Due to private information, not all gains from trade are realized, however. The rent extraction-efficiency trade off typical of problems involving asymmetric information induces incomplete liquidation.

Thus, the model predicts that in the presence of private information on positions, some longs liquidate prior to expiration, whereas some abstain from early liquidation in favor of playing the delivery end game. Big longs hold out, but some smaller-to-medium sized longs liquidate early.

The model has implications for the behavior of prices over time.

First, if the large long hits the large short's bid, P exceeds the post-trade price. To see this, note that if the large short trades at t_0 , atomistic shorts and longs are willing to trade at t_1 at a price equal to the expected price in the delivery end game. This price is $\max[P_c, \mathcal{A} + \beta E(x|x \leq x^*) - \beta S]$. Further,

$$P = \mathcal{A} + \beta x^* - .5\beta S > \max[P_c, \mathcal{A} + \beta E(x|x \leq x^*) - \beta S]$$

Thus, the price falls after a large long liquidates some or all of his position before the end game. The fact that the price sometimes declines implies that the small shorts free ride off the large short.

Second, if the large long does not hit the large short's bid, at t_1 the atomistic longs and shorts are willing to trade at a price equal to the expected price in the delivery end game: $\mathcal{A} + \beta E(x|x > x^*) > P$.⁹

Third, since the t_1 prices are based on the conditional expectation of x , prices may either rise or fall from t_1 to t_2 regardless of whether the long hits the bid at t_0 because the actual value of x is revealed at t_2 through the large long's bid.

Thus, in the model prices fluctuate even though by assumption there are no fundamental shocks to supply and demand. Prices can exhibit continuations up, continuations down, or reversals. Moreover, supercompetitive prices can appear prior to the end game, and persist for some time. All of these fluctuations are merely technical in nature, and unrelated to any supply and demand changes (i.e., changes in θ_D , θ_S , ϕ_D , and ϕ_S .) These technical fluctuations, which are purely a consequence of market power, strategic

⁹If the large long can trade at t_1 , due to the winner's curse problem no atomistic short is willing to trade at t_1 . Then, the market merely liquidates at t_2 at the end game price.

behavior, and private information about positions, inject noise into futures prices. Thus, manipulation and pre-manipulation bargaining interfere with the price discovery functions of the market in the pre-expiration period.

A simple example illustrates some other possible outcomes of the bargaining process with a large short. In the example, x is distributed uniformly on the interval $[S, I]$. Moreover, $S > 2\hat{x}$; that is, the short's position is more than twice as large as the minimum long position that is required to squeeze the market. Therefore, by (3), $\frac{S}{2} = x^* - S$, so $x^* = \frac{3}{2}S$, and $P = \mathcal{A} + \beta S > \mathcal{A} + \beta\hat{x}$. Thus, the large short liquidates at a supercompetitive price. Moreover, though some longs liquidate, those with $x \in [S, x^*]$ do not liquidate entirely, leaving them with a position of $x - S$. Consider the post-partial-liquidation position of a long with a position of x^* . Here $x - S = .5S$. If $.5S > \hat{x}$, this long can still squeeze at delivery, although this squeeze is not as severe as would occur absent a pre-expiry partial liquidation. Given the assumption that the large short's position is more than twice as large than the minimum position the long requires to squeeze, this condition holds. So, one possible outcome of the liquidation process is for the short to liquidate at a supercompetitive price; for the price to fall subsequent to this liquidation; but for the price at expiration to remain above the competitive price.

Other assumptions about the distribution of the long's position implied by the short's signal generate other interesting results. For instance, if the distribution of the long's position is conditionally normal, with the underlying normal distribution having mean 150 and variance 50, and the support of the conditional normal $[S, 600]$, a large short with $S = 120$ chooses $x^* = 168.7$. As before, if $S > 2\hat{x}$, the liquidation occurs at a supercompetitive price; the price falls after the short buys back his position; and there is

still a squeeze at expiration. It should also be noted that the short's cost of liquidating his position is smaller, the smaller the variance of the underlying normal. Thus, the short has an incentive to buy a more precise signal. This explains the development of commercial intelligence systems by large shorts, and their willingness to employ brokers who provide information about the activities of other market participants.

The examples also demonstrate that the large long's share of open interest is a misleading measure of his market power. In the normal example, for instance, if the large long's true position is actually 165, his market share is approximately $1/3$, but he still successfully liquidates a fraction of his position at a supercompetitive price by trading with the large short. Indeed, increasing open interest in the normal example has virtually no impact on x^* , and hence on P . The irrelevance of market share also obtains at expiration, as the competitive fringe of small longs merely liquidates at the price the long chooses, and his choice does not depend in any way on the size of this competitive fringe.¹⁰ In the model, the atomistic traders may liquidate positions after the large long and short have an opportunity to trade, which means that one outcome in the model is for the long's share of open interest to increase as the contract approaches expiration even if he liquidates a portion of his position prior to expiration. It is the size of the position— x —that matters, not market share.

It should also be noted in these examples, it is an equilibrium for no atomistic longs to hit the large short's bid even if they are guaranteed having their order executed. Note that under this assumption, the atomistic long

¹⁰The Donaldson-Cooper model also implies that market share is irrelevant to the long's market power, and that the size of his position relative to deliverable supply is what matters.

receives P if he hits the bid if guaranteed an execution. If he does not hit, he receives $P_m(x)$ if $x > x^*$, and $P_m(x) - \beta S$ for $x \leq x^*$. Thus, if

$$P < E[P_m(x)] - \beta S F(x^*)$$

each individual long has no incentive to hit the large short's bid. Substituting for P in terms of x^* implies that this condition holds if:

$$S(F(x^*) - .5) < E(x) - x^*$$

In the normal example, $E(x) = 172.96$, $x^* = 168.7$, $F(x^*) = .51$, and $S = 120$, so this condition holds. Note that since each small long faces the adverse selection problem described above, each might not hit the bid even if this condition does not hold.

In sum, the model of this section demonstrates that private information on a large long's position can lead to rich dynamics in the period leading up to contract expiration when there is a large short. The large short has an incentive to bid to buy back his position at lower price than he expects to pay in the end game, which is still above the competitive price. Some longs take the large short's bid even though it is below the price they expect to extract during the end game because the average revenue per contract the long receives is lower than the average price that the short pays during the endgame due to the deadweight losses of manipulation and the effect of burying the corpse. The large short chooses his bid to trade off rent extraction against the gains from trade. Due to this trade off, partial liquidation sometimes, but not always, occurs prior to the end game.

The model predicts that large longs sometimes liquidate portions of their positions prior to expiration even though this dilutes (and may eliminate) their market power in the end game. Moreover, the model predicts fluctuations in prices as the contract nears expiration that are driven purely by

technical factors, and are completely unrelated to fundamentals. The model also implies that this pre-expiration liquidation mitigates the deadweight costs of manipulation.

Although the private information in the model relates to positions, one can imagine other (private) differences in the “types” of longs that lead to similar results. For instance, some longs may have different costs of mounting a legal defense against manipulation charges (or believe their costs differ); longs with high costs are willing to liquidate at lower prices than longs with high defense costs. As another example, the reputational cost of manipulation may differ across traders. For instance, Yasuro Hamanaka, the copper trader for Sumitomo who manipulated the copper market in 1995-1996, was facing increasing difficulties of concealing his off-book trading losses, and needed to make a quick profit to recoup some of these losses; given his short time horizon, Hamanaka was much more likely to discount the future reputational consequences of manipulating than other traders not facing such desperate straits. Similarly, traders may have private information about internal control mechanisms and their compensation structures that make some more willing to manipulate than others. To the extent that information on these differences is private, they can influence pre-expiration bargaining between longs and shorts.

5 Private Information About Delivery Costs

Many would-be manipulators have been the poster children for the old joke: “Want to make a small fortune trading commodities? Start with a large one.” One of the most common ways that manipulators have met their ruin is to be buried in an avalanche of deliveries. Several colorful examples illustrate the point. In 1892, speculators Coster and Martin demanded \$1

per bushel to liquidate their large long position. Rather than hitting the bid, grain elevator operators short futures assiduously shipped and conditioned massive quantities of corn that they dumped on the unsuspecting would-be cornerers (Taylor, 1917). A few years later, in 1898, P. D. Armour used ice breaking tugs to bring unexpectedly large supplies of wheat to break Joseph Leiter’s wheat corner (Taylor, 1917).¹¹ More than 80 years later, the Hunts were deluged with deliveries of silver, much of it originating in the trousseaus of Indian brides, which forced them to liquidate huge long silver positions.

These types of events are readily understood when one incorporates private information about delivery costs into the basic end game model. Consider a model in which shorts have superior information about delivery costs than the large, cornering long. This is plausible, inasmuch as shorts are often large commercial firms with extensive networks of buyers and handlers of the commodity. A large firm such as Cargill, for instance, has numerous elevator and processing facilities located throughout the grain belt which provide the firm with extensive information about supply conditions. The grain elevator operators who thwarted Coster and Market also had intimate knowledge of how much corn could be obtained at what price.

Formally, I assume that shorts know the intercept of the supply curve, θ_S , but that the cornering long only observes a noisy signal. The density of the true θ_S conditional on the long’s signal is $g(\theta_S)$. This density has support $[\theta_{SL}, \theta_{SH}]$.

During the end game, the long makes an all-or-nothing offer of \mathcal{P} for his position of x contracts. Shorts then decide how many contracts to liquidate at that price, and how many deliveries to make.

¹¹The Leiter corner was the model for Frank Norris’s realistic novel, *The Pit*.

Knowing θ_S , shorts deliver to the point that the marginal cost of delivery equals \mathcal{P} . This results in shorts making $Q(\mathcal{P}, \theta_S)$ deliveries:

$$Q(\mathcal{P}, \theta_S) = \frac{\mathcal{P} - \theta_S}{\phi_S}$$

The long chooses \mathcal{P} to maximize his expected profit:

$$V = \max_{\mathcal{P}} \int_{\theta_{SL}}^{\theta_{SH}} \Pi(Q(\mathcal{P}, \theta_S), \mathcal{P}) g(\theta_S) d\theta_S$$

where

$$\Pi(Q(\mathcal{P}, \theta_S), \mathcal{P}) = Q(\mathcal{P}, \theta_S)[\theta_D - \phi_D Q(\mathcal{P}, \theta_S)] + [x - Q(\mathcal{P}, \theta_S)]\mathcal{P}$$

The first order condition is:

$$0 = \int_{\theta_{SL}}^{\theta_{SH}} \left[\frac{\partial \Pi}{\partial Q} \frac{dQ}{d\mathcal{P}} + \frac{\partial \Pi}{\partial \mathcal{P}} \right] g(\theta_S) d\theta_S$$

where $\partial \Pi / \partial Q = 1 / \phi_S$ and $\partial \Pi / \partial \mathcal{P} = -Q$.

Given the linearity of the demand and supply functions, it is straightforward to show that the profit maximizing \mathcal{P} is of the form:

$$\mathcal{P} = B_D \theta_D + B_x x + B_S E(\theta_S)$$

where $B_S > 0$. That is, the large long chooses the number of deliveries to take based on his expectation of the location of the supply curve. It is also possible to show that the price the large long chooses equals the *average* price he would choose given complete information on θ_S . Evaluated *ex post* (once θ_S is revealed), however, sometimes this price is too high, and sometimes it is too low. As a result, the profitability of manipulation is lower when information is incomplete.¹²

¹²Pirrong (2008) shows that the profitability of manipulation is a convex function of θ_S . Therefore, due to Jensen's inequality, his profit at the average price in the incomplete information game is smaller than his average profit when he can vary price with supply conditions.

If the large long is overly bullish on supply, that is $E(\theta_S) < \theta_S$, he receives more deliveries than he expects. Indeed, if he is very bullish, he takes far more deliveries than he expects and will suffer large losses because (a) he liquidates few contracts at the price he demands, and (b) receives large quantities of deliveries that he must sell at the depressed post-delivery price (which is less than P_c).

The analysis therefore implies that asymmetric information about supply conditions is a deterrent to manipulation. It therefore suggests that commercial traders with information about supply conditions similar to that possessed by commercial shorts are more dangerous cornerers than traders who lack such knowledge.

6 Step Up Orders

Several manipulation cases involve the use of step up orders at expiration by a large long. That is, the large long offers contracts at a particular price; once those orders are filled or rejected, he offers additional contracts at a higher price; raises his offer again as those orders are filled or rejected, and so on. Cases involving the use of such orders include Cargill, Great Western, Indiana Farm Bureau, and British Petroleum. Indeed, courts and regulators often consider the use of step up orders highly suspicious, manipulative behavior.¹³

At first blush, the use of step up orders is hard to understand. If there is an active, open, and transparent market for the deliverable good, all shorts

¹³The fact that so many manipulation cases involve step up orders may therefore arise due to selection bias; if regulators consider step up orders a badge of manipulation, cases brought are more likely to involve such behavior even though a large long can manipulate without the use of such orders. Johnson and Hazen (1997) state “it appears that step-up orders will attract the attention of the reviewing tribunal.”

should be willing to pay the marginal cost to deliver Q units to liquidate their positions. That is, under these circumstances all shorts are homogeneous and market prices communicate information on the marginal cost of delivery. This makes the use of step up orders problematic. If a manipulating long sells any contracts at a given price, his market power declines, and his profit maximizing price for his remaining contracts falls.

To see this, assume that an instant before trading ends, a trader long x contracts initially chooses a price $P^* < P_m(x)$ in the expectation of charging a higher price at the last instant of trading. If a short actually buys at this price, the long will only be able to sell at the end of trading at a price *lower* than $P_m(x)$. Thus, the long is worse off by choosing an initial trading price less than $P_m(x)$ and definitely finds it unprofitable to step up subsequent offers if this initial offer is lifted. Conversely, choosing a price greater than $P_m(x)$ is not profitable either, as by assumption $P_m(x)$ is the profit maximizing take-it-or-leave it price.

Introducing some heterogeneity and private information among shorts can make step up orders rational, however. Assume that short i incurs marginal cost of delivery C_i ; that the number of deliveries one short makes has no impact on other short's marginal cost of delivery; and that C_i is private information to each short. Under these assumptions, the large long is exactly analogous to a monopoly seller facing customers with different, and private, reservation prices.

It is well known that the monopoly seller in this case can maximize profits by discriminating among the different buyers by offering a menu of contracts (Laffont and Martimort, 2002). These contracts specify a quantity q and a transfer $T(q)$. Using the revelation principle, there is a direct mechanism that induces the buyers to reveal their C_i , and choose a $q(C_i)$ and $T(q(C_i))$.

The monopolist could offer this menu of contracts using step up orders (or step down orders, for that matter.) For instance, the long could offer via open outcry (or on a computer screen) a price-quantity pair that a short with $C = C'$ should take. If there is such a short, this offer will be accepted. Regardless of whether it is accepted or not, the long then offers a price-quantity pair tailored to screen a short with $C = C'' > C'$; this will in general involve a higher price, so this process produces step up orders.

The linkage of the transfer and the quantity sold is necessary to satisfy the incentive compatibility constraints. The contracts satisfying these constraints must satisfy monotonicity constraints. That is, $q(C'') > q(C')$ for $C'' > C'$. This is necessary to induce truthful revelation; if it did not hold, a short with a small delivery cost would like to mimic shorts with higher delivery costs and thereby liquidate a greater portion of his position than is desirable for the long. Moreover, shorts with high (low) C 's should pay high (low) prices. This induces an outcome that mitigates, but does not eliminate deadweight costs; high delivery cost shorts pay higher prices to liquidate, but make fewer deliveries, and most deliveries are made by the more efficient shorts.

There is data from some manipulation cases that permit testing for this monotonicity condition. In the Cargill case, this grain firm submitted stepped up offers at the very close of the May, 1971 wheat contract on the CBOT. The firm offered 200,000 bushels at 227 cents/bu, 200,000 at 227.25, 300,000 at 227.5, 400,000 at 227.75, 500,000 at 228, and 390 at 228.25. Except for the second and last orders, these offers did exhibit monotonicity; higher quantities were offered at higher bids.

Two other cases involving step up orders do not satisfy monotonicity. In the Indiana Farm Bureau case, the firm offered 100,000 bushels of corn

at 370 cents/bu, 100,000 at 375, 100,000 at 380, 100,000 at 385, and 90,000 at 390. In British Petroleum's corner of the propane market, the firm sold lots of propane at successively higher prices, in increments of a half-cent per gallon, but each lot was the same size—25,000 gallons.

Thus, although a large long can use step up orders to price discriminate among shorts with different delivery costs and private information about those costs, the limited empirical examples of the use of these orders provides at best mixed support for this interpretation. Moreover, it should be noted that the assumption of differences in delivery costs across shorts presupposes some impediment to trade among them. In the standard end game model with no asymmetric information, trade in the cash markets would ensure that (a) the cost of delivering Q units is minimized, and hence (b) the marginal cost of delivery is equalized across shorts. The equation of marginal costs across shorts is inconsistent with the assumption of the model used here to rationalize the use of step up orders. One possibility is that the very information asymmetries that lead to the use of step up orders as a means of price discrimination impedes trades among shorts.

7 Summary and Conclusions

Corners and squeezes are, and have always been, regular features of derivatives markets. Although research has identified the factors that make market power manipulations possible—namely, an upward sloping marginal cost of delivery in the range of the position held by a large long—and the consequences of manipulation, the received bare-bones models cannot capture the richness and complexity of trading as a contract nears expiration. Herein I show that a simple and straightforward addition to these models can generate interesting trading dynamics and solve some puzzling aspects of his-

torical corners. Specifically, asymmetric information can greatly affect the dynamics of trading in derivatives contracts that are vulnerable to a squeeze.

The most important results relate to the effect of private information concerning the cornerer's position. The deadweight losses and wealth transfers that occur during the delivery end game provide a motive for longs and shorts to liquidate prior to expiration and avoid the losses associated with playing the end game. If shorts do not know the cornering long's precise position, however, the information asymmetry can impede these mutually beneficial trades.

When all shorts are atomistic, the results are stark: free rider and coordination problems preclude the consummation of any mutually beneficial trades. The presence of a large short can mitigate these problems, however. The large short is always willing to bid a price to liquidate his positions that is sufficiently high to induce some longs that would otherwise corner the market in the end game to sell contracts prior to the end of trading. The rent extraction-efficiency trade off common to contracting in the presence of asymmetric information affects the short's bid, however, meaning that not all mutually beneficial trades are completed.

The model predicts that some longs sell their cornering interests prior to the end game; that some longs do not and instead play the end game despite the costs associated therewith; that some longs sell part of their positions, but still retain sufficient market power to squeeze during the end game, though the resulting squeezes are less severe than what would occur in the absence of the pre-delivery partial liquidation. The model also predicts the pre-expiration liquidation of large long positions at supercompetitive prices, and that prices in pre-delivery trading fluctuate (exhibiting continuations up, continuations down, and reversals) even in the absence of

the arrival of fundamental supply and demand information; such “technical fluctuations” are the result of the interaction of market power and private information. The model also implies that the structure of the short side of the market—the sizes and concentration of short positions—influences the nature and efficiency of trading during a corner, and (perhaps counterintuitively) that the presence of large shorts actually make a cornering long better off.

Private information on the supply side can also shed light on other noted features of corners and squeezes. For instance, private information on delivery costs can explain why some corners result in huge deliveries that impose large losses on the cornerer. Private information about delivery costs, and heterogeneity of these costs between shorts, may also motivate the otherwise puzzling use of “step up” orders near expiration as a means of price discrimination, although it should be noted that the actual use of such orders in manipulations is not uniformly consistent with the predictions of these price discrimination models.

In sum, the incorporation of private information into models of market power in derivatives markets resolves some important questions about the trading process as a derivatives contract approaches expiration.

A Equilibrium With a Menu of Contracts

The model in the main text assumes that the large short chooses a single offering price for the S contracts he owns. In general, this trader can do better with non-linear pricing schemes. For instance, the large short could offer a menu of contracts $\{q(x), T(x)\}$ where $q(x)$ is the quantity of contracts offered the large long with x contracts, and $T(x)$ is the transfer paid from the large short to this large long. In a derivatives market, the large short

could offer this menu through a set of all-or-nothing orders, perhaps offered sequentially in an open outcry or computer auction.

The revelation principle implies that under certain conditions, there exists a direct mechanism whereby the large longs truthfully report their “types,” i.e., their positions x . In this direct mechanism, the contracts offered by the short must satisfy the relevant self-selection and participation constraints. In the present model, since x is continuous, satisfaction of local self-selection constraints implies the satisfaction of global constraints.

A crucial condition is the so-called single-crossing property, which in this model describes a property of the utility functions of the large longs. A large long with position x receiving a contract pair $\{q, T\}$ receives wealth (i.e., utility) over and above his reservation level of:

$$U(x, q, T) = \Pi(x - q) + T(q) - \Pi(x)$$

The large long sells q contracts to the short, and receives $T(q)$ in return. In the end game the long liquidates the position $x - q$ and receives $\Pi(x - q)$.

Note:

$$\begin{aligned} \frac{\partial U}{\partial T} &= 1 \\ \frac{\partial U}{\partial q} &= -\Pi'(x - q) = -P_m(x - q) \end{aligned}$$

Thus, the marginal rate of substitution (the slope of the large long’s indifference curve) is:

$$MRS = -\frac{\partial U/\partial q}{\partial U/\partial T} = P_m(x - q)$$

The single crossing property requires that this marginal rate of substitution increase with x . Note:

$$\frac{\partial}{\partial x} \left[-\frac{\partial U/\partial q}{\partial U/\partial T} \right] = \frac{\partial P_m(x - q)}{\partial x} > 0$$

Therefore, the single crossing property holds.

The short's objective function is to minimize the expected cost of liquidating his S contracts. This is equivalent to maximizing:

$$\int_{x_0}^I [-(S - q)P_m(x - q) - T(q)]f(x)dx$$

The $(S - q)P_m(x - q)$ is the cost that the large short incurs during the end game if the large long liquidates q contracts at t_0 , and $T(q)$ is the consideration the large short pays the large long at t_0 . Since $T(q) = U - \Pi(x - q) + \Pi(x)$, this is equivalent to:

$$\int_{x_0}^I [-(S - q)P_m(x - q) - U + \Pi(x - q) - \Pi(x)]f(x)dx$$

The large short maximizes this objective function subject to the incentive compatibility and participation constraints. Consider the IC constraint. A long with position x who chooses to report his position as \tilde{x} receives utility:

$$U(\tilde{x}) = T(\tilde{x}) + \Pi(x - q(\tilde{x})) - \Pi(x)$$

The first order condition for this problem must hold at $\tilde{x} = x$:

$$T'(x) - P_m(x - q(x))q'(x) = 0 \quad (4)$$

The second order condition must also hold at $\tilde{x} = x$:

$$T''(x) + P'_m(x - q(x))[q'(x)]^2 - P_m(x - q(x))q''(x) \leq 0 \quad (5)$$

Differentiating (4) with respect to x produces:

$$T''(x) + P'_m(x - q(x))[q'(x)]^2 - P_m(x - q(x))q''(x) - P'_m(x - q(x))q'(x) = 0 \quad (6)$$

Substituting (6) into (5) implies:

$$P'_m(x - q(x))q'(x) \leq 0$$

Since $P'_m(x - q(x)) > 0$, this implies:

$$q'(x) \leq 0$$

This monotonicity constraint can be incorporated into the relevant optimal control problem.

Moreover, applying the envelope theorem:

$$\frac{dU}{dx} = P_m(x - q) - P_m(x) \leq 0$$

This is also relevant for the optimal control problem. In particular, it implies that the participation constraint $U(x) \geq 0$ is binding only for $x = I$ (or, for the largest x if the upper support of x is strictly less than the open interest.) Note particularly that $U(x_0) > 0$.

Given the foregoing, the Hamiltonian for this problem is:

$$H(q, U, \mu, x) = \mu(x)(P_m(x - q) - P_m(x)) + [-(S - q)P_m(x - q) - U + \Pi(x - q) - \Pi(x)]f(x)$$

where μ is the co-state variable. The Pontryagin principle implies:

$$\frac{d\mu}{dx} = -\frac{\partial H}{\partial U} = f(x)$$

The transversality condition implies:

$$\mu(x_0) = 0$$

Therefore:

$$\mu(x) = F(x)$$

Since $d\Pi(x)/dx = P_m(x)$, the first order condition is:

$$[P_m(x - q) + (S - q)P'_m(x - q) - P_m(x - q)]f(x) - \mu(x)P'_m(x - q) = 0$$

Therefore:

$$(S - q)P'_m(x - q) = P'_m(x - q)\frac{\mu(x)}{f(x)}$$

Thus:

$$q = S - \frac{F(x)}{f(x)}$$

There are several noteworthy features of this solution. First, when $x = x_0$ (its lower support), $q = S$. Second, if $F(x)/f(x)$ is increasing in x (as is the case for most parametric single peak densities), q is decreasing in x . Thus, q satisfies the monotonicity condition that is necessary for this problem.

In this solution, the large short offers a menu of contracts such that all longs except the largest earn a profit from early liquidation that exceeds the profit he would have earned by playing the end game. That is, the large short cannot extract all the rents despite the assumption that he has all the bargaining power since he offers the menu of all-or-nothing contracts. The short must leave the long with some rents in order to ensure incentive compatibility; private information prevents the short from extracting all rents.

There are some implausible implications of this model, however, as compared to those of the model in the main text. Note in particular in the solution of this problem, it may be the case that $q(x) < 0$ for some x for arbitrary S and $f(\cdot)$. Since $q > 0$ is the number of contracts *bought* by the large short, a negative value implies that the short *sells* additional contracts. This is necessary to ensure incentive compatibility. The short is compensated for these additional sales through a high transfer. Specifically, consider $x = I$. The individual rationality constraint implies that $U(I) = 0$. Therefore:

$$T(I) + \Pi(I - q(I)) = \Pi(I)$$

Since $\Pi(I - q(I)) > \Pi(I)$ for $q(I) < 0$, $T(I) < 0$. This indicates that the large long pays the short an amount for the additional contracts that just

equals the additional manipulative profit that these contracts allow him to extract.

Thus, one outcome of this model is for small-to-medium longs to reduce their positions, but for large longs to increase them. This further implies that pre-expiration bargaining with price discrimination makes the deadweight loss of manipulation worse in those cases where the very large long increases his position. This does not seem particularly plausible, nor are there any ready-to-hand historical examples of this conduct. Furthermore, the fact that the short's strategic behavior sometimes makes the end game more severe may affect the behavior of the small shorts. Free riding is no longer guaranteed because atomistic shorts are sometimes worse off as a result of the bargain between the large long and short when the latter uses a menu of contracts, whereas they are always better off waiting for the end game when the large short makes a single take or leave it offer. This raises the possibility that this set of contracts is not supportable in the market in the face of competition from the atomistic shorts.

The price implications of this model are somewhat different from those of the model in the main text. In particular, the long reveals his type through his choice of contract. Thus, whereas in the model in the main text, there may or may not be trade at t_0 , and prices may fluctuate from t_1 to t_2 (since the outcome of trade at t_0 does not completely resolve uncertainty about the long's type), in this trade occurs almost surely at t_0 , and price does not change from t_1 to t_2 as there is no uncertainty about the long's type after t_0 .

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